A New Family of Variational-Form-Based Regularizers for Reconstructing Epicardial Potentials from Body-Surface Mapping

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The potential-based inverse electrocardiographic problem aims to noninvasively recover epicardial potentials from body-surface recordings,with the torso volume conductor typically modeled using the finite element or boundary element method. This inverse problem is intrinsically ill-posed and commonly solved by the Tikhonov regularization, where discrete operators are needed to constrain certain properties of the epicardial potentials (amplitude, gradient or curvature). We propose a new family of regularizers within the classic Tikhonov framework, using the continuous basis functions and the variational principle underlying finite element methods. These variational-formed regularizers constitute an alternative to the traditional Tikhonov regularizers, but have four major advantages. First, if the discrete inverse problem is derived from the governing equations by finite element methods, the variational-formed regularizers automatically conform to certain variational principles inherently assumed by the discrete problem. Second, while traditional regularizers consider the discrete Euclidean norm, the variational-formed regularizers consider the L2-norm defined over a continuous domain, independent of discretization resolution, thereby achieving consistent regularization under multi-scale simulations. Third, the variational formulation enables an explicit matrix form of the discrete gradient operator over irregular meshes, which is difficult to obtain from traditional discretization. Fourth, the variational formulation allows simultaneous imposition of multiple constraints efficiently.

We validated the efficacy of the variational formulation through finite element simulations on a phantom experiment involving a live canine heart suspended in a human-torso-shaped electrolytic tank. When the finite element model was undergoing refinement, the variational-formed regularization maintained consistent regularization and improved reconstruction of epicardial potentials, whereas the traditional Tikhonov worsened the reconstruction. The variational-formed gradient regularizer notably outperformed the zero-order Tikhonov. The gradient regularizer also effectively captured epicardial potential features arising from ischemia. These early results suggest that the variational formulation may provide new improvements to the simulation of a broader range of potential-based electrocardiographic inverse problems.