Theoretical and Experimental Reflection Coefficients in Flexible Tubes as a Function of the Mach Number

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Abstract

The standard formulation of Wave Intensity Analysis (WIA) assumes that the flow velocity (U) in the conduit is negligible with respect to the velocity of propagation of waves (c) in the system. In large conduit arteries, U is relatively high due to ventricular contraction and c is relatively low due to the high compliance; thus M=U/c is significantly greater than 0. Therefore, the aim of this study is to identify experimentally the relationship between M and the reflection coefficient in vitro.

Combinations of flexible tubes, of 2 m in length with circular cross sectional area, isotropic and uniform material properties along their longitudinal axes were used to present a mother connected to a daughter tube. An approximately semi-sinusoidal pulse was generated at the inlet of each tube using a syringe pump, first, the waves speed was determined using the foot to foot and PU-loops methods in the condition of unperturbed velocity ($U_0=0$). The theoretical reflection coefficient (R_t) for M=0 has been calculated. Then, superimposing steady flow using constant DC motor over the pulse waveform generated by a syringe pump, we recorded simultaneously pressure and velocity in the mother tube to identify the relationship between M and R. WIA was used to separate the pressure waveforms and the experimental reflection coefficients (R) at M>0 were determined as dP-/dP+.

In our experiments of pulsatile flow in long flexible tubes, R increased significantly with small values of M (order of 10^{-2}). In the range of M=0 to 0.02, R increases by 4% to 36%. Further, the function R(M) changes significantly with the geometrical and mechanical features of the connected tubes.

1. Introduction

Arterial reflected waves play an important role in determining the features of the arterial pressure waveforms, including systolic pressure (1, 2). Moreover, arterial aging and pathologies (e.g. hypertension) affect both the magnitude and timing of arterial reflected waves and, therefore, the features of the pressure waveform (3, 4).

The magnitude of reflected waves can be quantified by means of the reflection coefficient; i.e. the ratio between the backward and forward component of the pressure waveform or of the wave intensity. This parameter is normally assumed to be independent of the mean value of blood velocity in the artery. In fact, to derive the standard WIA formulation the blood velocity is assumed to be much lower than the waves speed in arteries (M≈0), and, therefore, the convective component is neglected (5). While this assumption is reasonable in a considerable portion of the arterial tree, large conduit arteries, such as the aorta or the pulmonary artery, present a relatively low wall stiffness and relatively high blood velocity. In these regions, neglecting the convective term may induce errors in the estimation of wave reflections. The aim of the present work consists in determining the effect of the superimposition of a steady state flow to a pulse waveform on the reflection coefficient in flexible tubes.

2. Methods

2.1. Experimental setup

The experimental setup is described in Fig. 1.

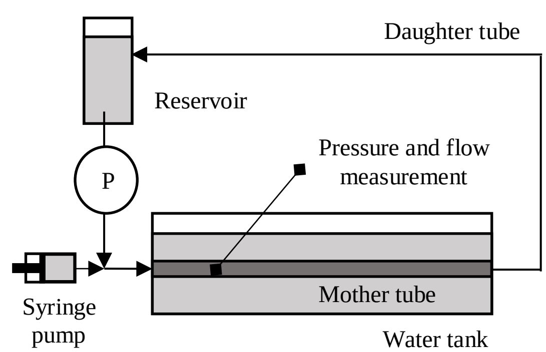

Figure 1 - Schematic representation of the experimental setup. Pressure was recorded using a transducer tipped catheter, while flow using an ultrasound probe. All the components were located on a horizontal plane.

2.1.1. Tubes

For the present work, N=6 2 meters long mother tubes and N=4 >9 meters long tubes of different size and material have been used. The length of the daughter tube was necessary to separate in terms of timing the reflection generated by the connection between mother and daughter tubes by the one caused by the connection between daughter tube and reservoir. Information on tubes is provided in table 1. All the tubes were used as mother tube, while only D_{in}=8, 10, 17 and 20.6 were used as daughter tube. c has been calculated performing preliminary experiments on each tube and averaging the results obtained with the foot-to-foot and PU-loop techniques. The geometrical and mechanical properties of each tube were uniform along its longitudinal axis.

The connection between mother and daughter tubes was realized overlapping them of approx. 1-2 cm. If the difference in diameter was too high, a third tube was inserted between mother and daughter to allow the connection.

Table 1 - Properties of the tubes used in the experiment, D_{in}: internal diameter, h: wall thickness, c: waves speed.

D_{in} [mm]	**h** [mm]	**Material**	**c** [m/s]
8	2	Silicone	25.02
9.525	0.8	Latex	12.70
10	1	Silicone	21.52
17	1.5	Rubber	20.76
19.05	2.4	Latex	14.17
20.6	1.7	Rubber	20.38

2.1.2. Pumps

Two pumps were used for this experiment: a piston pump, producing an approx. single semi-sinusoidal pulse, and a centrifugal pump providing a continuous flow. The speed of the centrifugal pump was adjusted with a knob in order to vary the flow and characterize the variation of R as a function of M.

2.1.3. Measurements

Simultaneous pressure and flow measurements were performed at the same axial location (52 cm from the inlet of the mother tube). Pressure was measured using a 6F pressure transducer tipped catheter (Millar Instrument, Inc, Ithaca, NY, USA), and flow was measured using ultrasound flow probes (Transonic System, Inc, Ithaca, NY, USA) matching the diameter of the mother tubes.

2.2. Theoretical reflection coefficient

When M≈0, the R_t coefficients characterizing the connection between a mother and a daughter tube is calculated as follows:

$$R_t = \frac{\frac{A_1}{c_1}-\frac{A_2}{c_2}}{\frac{A_1}{c_1}+\frac{A_2}{c_2}} \qquad (1)$$

where A_1 and c_1 are the internal sectional area and waves speed of the mother tube, respectively, and A_2 and c_2 of the daughter tube.

2.3. Experimental reflection coefficient

The standard formulation of WIA was used to separate the pressure and velocity waveforms in their forward and backward components (6). The experimental reflection coefficient (R) was calculated as the ratio between the first peak of the backward and forward component of the pressure waveform:

$$R = \frac{dP_-}{dP_+} \qquad (2)$$

2.3. Data analysis

For each connection configuration, pressure and velocity waveforms were recorded 7 times for each M value. The rotational speed of the centrifugal pump was varied between 0 and the maximum flow (which depended on the resistance provided by the mother and daughter tubes connected downstream). The whole range of flows was divided into 6-8 steps and P and U recordings were performed for each step.

Collected data were analyzed with a Matlab code (The Mathworks, Natick, MA, USA). The value of the reflection coefficient for each M was calculated averaging the results obtained in the 7 repeated measurements.

3. Results

3.1. Reflection coefficient at M=0

Table 2 shows a comparison between R_t calculated with (1) and R_0 (R when M=0) determined experimentally for each connection configuration used in the present work.

3.2. Reflection coefficient a function of M

With all the connection configurations, an increase of R was found as M was increased (figure 2). This was found for both negative and positive R_t. A linear fitting of R(M) provided an R^2>0.95 in all the connection configurations, except for connection G (R^2=0.92). Interestingly, the percentage of increase of R at a given M was different in different connection configurations (as indicated by the slope column in table 2). For example, at M≈0.02, R increased by 50% in connection D, by 30% in H and by 10% in connection F.

Connection	Mother Ø [mm]	Daughter Ø [mm]	R_t	R_0	Slope (95% C.I.)
A	20.6	8	0.78	0.696 (±0.013)	8.7 – 25.9
B	17	8	0.697	0.694 (±0.010)	12.3 – 14.0
C	19.05	8	0.817	0.761 (±0.011)	10.9 – 16.3
D	10	8	0.290	0.285 (±0.005)	5.6 – 8.8
E	17	10	0.499	0.505 (±0.004)	6.3 – 8.2
F	19.05	19.6	0.105	0.076 (±0.011)	2.8 – 4.5
G	9.525	10	0.212	0.206 (±0.006)	2.1 – 2.7
H	9.525	8	0.473	0.531 (±0.008)	2.2 – 3.4
I	10	19.6	-0.635	-0.605 (±0.009)	2.1 – 3.4
J	10	17	-0.499	-0.465 (±0.008)	2.1 – 3.7

Table 2 - Comparison between the theoretical reflection coefficient R_0 and the experimentally determined R at M=0. R is expressed as mean (SD).

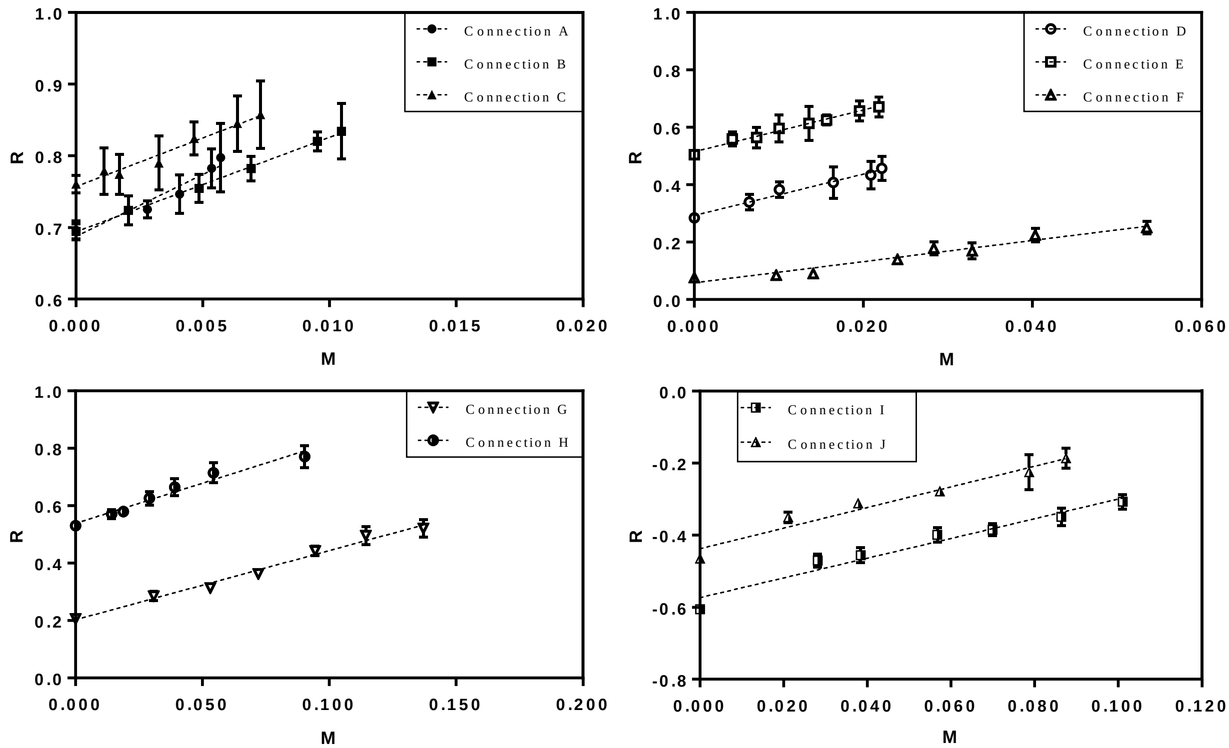

Figure 2 - R as a function of M for all the connection described in table 2. Data are presented as mean (SD).

4. Discussion

The results showed a good agreement between the theoretical reflection coefficient and the experimental reflection coefficient at 0 flow. The largest error was found for connection A (R_t=0.78 and R_0=0.70). The theoretical reflection coefficient assumes an ideal connection between tubes with no overlapping. On the other hand, the real connection between tubes may have introduced a deviation of R_0 from R_t.

All the connections used in this set of experiments showed an increase in the value of the reflection coefficient with increasing value of Mach number. However, this finding has different implications for positive and negative reflection. In fact, while in the case of positive reflections the superimposed steady state flow enhances the discontinuity given by the connection between mother and daughter tubes, the discontinuity of negative reflections appears to be smoothed by increasing M with R approaching 0. Therefore, positive and negative reflections present an opposite behavior with increasing M.

Interestingly, the slope of the linear fitting line was not equal in different connections, while it seems to depend on the relative geometrical and mechanical properties of the connected tubes. These results seem to indicate that higher values R_0 produce higher slopes of R as a function of M. However, connection H shows a significantly lower slope with respect to D and E but has similar R_0, and comparable to connection F and G ($R \leq 0.2$ for M=0). Therefore, other elements seem to play a role in determining the trend of R as a function of M. For example, connection A, B, C, D, and E were made between silicone or rubber tubes having similar values of wall stiffness (c=20-25 m/s), while connection F, G, and H were between latex and silicone/rubber, with the first being significantly more compliant (c=12-15 m/s). This could possibly explain the difference between H and D-E. The slopes of the normalized reflection coefficient (R/R_0) have also been compared. A significant difference was found between all the connections (figure 3). In figure 3 (top), a different slope can be identified between connection A and E even for very low values of M ($\approx 10^{-2}$).

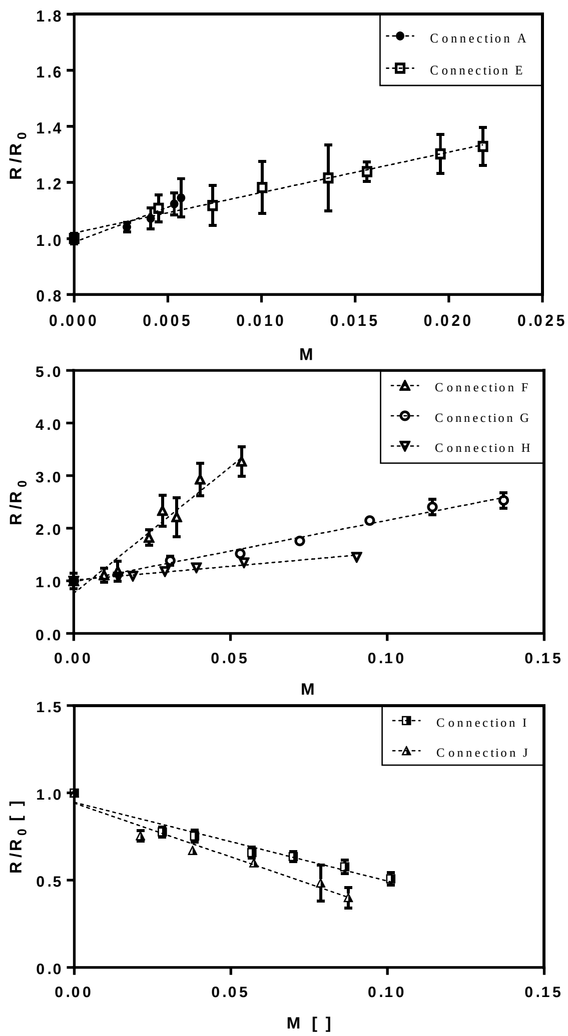

Figure 3 - Normalized reflection coefficient (R/R_0) as a function of M. Data are presented as mean (SD).

It is worth considering that R is bounded between -1 and 1 by definition, since $P_i = P_t - P_r$, where P_i is the incident pressure wave, P_t is the transmitted pressure wave and P_r is the reflected pressure wave. Therefore R(M) is expected to reach a plateau at high values of M. However, this trend was not shown in our results, except that for negative reflections. When fitting the results with second degree polynomial functions, the sign of the quadratic term was negative only in 4 of the connection configurations with positive R. This problem is due to the difficulties faced in reaching high values of M. In fact, the necessary long length of the daughter tube introduces a high hydraulic resistance, limiting the flow rate produced by the continuous flow pump.

5. Conclusion

In our experiments, the reflection coefficient is dependent on M, and a significant alteration of R can be obtained for M<0.05. Moreover, the function R(M) seems to be dependent on the relative geometrical and mechanical properties of the connected mother and daughter tubes. This could be relevant when considering waves reflections in large conduit arteries and the alteration of R with arterial pathologies.

More work is needed to define a mathematical formulation to describe the dependency of R from M and to explain the different behavior of different types of connections.

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