Invariant Electrical Mean Axis in Electrocardiogram

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Abstract

The mean electrical axis (MEA) is is an important measurement in clinical assessment. However, currently there is no agreement on a standardized calculation of the MEA. In order to standardize the MEA calculation, the following issues need to be addressed: 1) The net direction or net potential in the measurement of MEA needs to be defined. 2) Which underlying heart model which describes the electrical activity of the heart to use. 3) Which electrocardiogram (ECG) leads to calculate MEA from.

In present use for some applications, e.g. ECG-derived respiration, lead pairs I and III or I and aVF are used in axis calculation without further consideration of scaling and angle between the leads. This may cause an error up to $30°$.

The aim of the manuscript is to derive correct systematic equations for determining the heart vector and mean electrical axis (MEA) from any pair of leads in the hexaxial reference system. Comparison between 3 different definitions of net potential shows that the best expected standard deviation of $3.3°$ *between lead pairs' MEA are from net potentials that are defined to be area under the curve or sum of ECG voltages. Because of the invariant of MEA calculation the MEA from these two definitions have high fidelity.*

1. Introduction

The mean electrical axis (MEA) is the net direction (an angle) of the heart vector, dipole, [1] during depolarization and polarization processes. MEA is important because it correlates with the heart orientation, which is useful in clinical assessment [2]. MEA is often not specified, but often it refers to the net direction of the QRS complex [3–5] in the frontal plane, and not the actual direction in 3D space, in this case, of the ventricular depolarization.

Even though there is an ongoing standardization and interpretation of electrocardiogram (ECG)[6, 7], there is still no clearly defined and correct method to find the MEA. These issues can be divided into 3 parts.

The first issue addresses the definition of net direction of the heart vector. The direction of a dipole is determined from the voltages in the leads thus net direction can be translated to net potentials in the leads. An approach is to find the net potential as the sum of dominant QRS wave forms (V_{rs}) [4, 7, 8]. Another approach is to take the whole depolarization process into consideration, or a segment of it [2, 9–13], where net potential is the area under the curve (V_{area}) or sum of voltages of the segment (V_{sum}).

The second issue addresses the heart model, and how to find the dipole. In the classical homogeneous conducting sphere with a centric dipole model the voltage in the limb leads are dot products between the heart vector and the lead axes (unit vector)[6]. The leads form the Einthoven's triangle, and together with the augmented limb leads define the hexaxial reference system (figure 1). The Einthoven's triangle is in the frontal plane. However, in other models such as Burge and Frank the triangle formed by the limb leads are not in the frontal plane. For these models it is possible to project the dipole to the plane spanned by the lead vectors (triangle plane) and then project the dipole to the frontal plane to find the MEA. This of course induce an error because of the projection to the triangle plane first.

Thus it can be thought that a correct way is to find the dipole, and then project the dipole to the frontal plane. Methods to find the dipole directly from 12-leads ECG is to use inverse Dower or Kors transform[14, 15].

The third issue addresses the linear dependency between the limb leads and augmented limb leads. Ideally any combination of 2 of these leads will result in the same axis calculation. However, after preprocessing to enhance the signal quality the results might vary. This variation can be used as an indirect measurement of fidelity.

In present use for some applications, e.g. electrocardiogram-derived respiration[11, 12], lead pairs I and III or I and aVF are used in axis calculation without further consideration of scaling and angle between the leads. This can be considered a crude approximation, and may cause an error up to $30°$. Geometrically derived formulae exist[8, 16], but are inconvenient in usage.

The aim of the manuscript is to derive an equation set to calculate MEA with the Einthoven model which does not mistakenly cause errors mention above. Comparison

Table 1. Lead vectors, AHA coordinate system.

Lead	Triangle	x	y	z
I	Einthoven	1	0	0
II	Einthoven	0.5	$\frac{\sqrt{3}}{2}$	0
III	Einthoven	-0.5	$\frac{\sqrt{3}}{2}$	0
I	Burger	65	-21	17
II	Burger	25	120	-15
III	Burger	-40	141	-32
I	Frank	76	-27	14
II	Frank	30	146	-16
III	Frank	-46	173	-30

will be made between different definitions of net potential, V_{rs}, V_{area} and V_{sum}, and with reference axis calculations done by the equipment (Welch Allyn).

2. Dataset

The North Sea Race Endurance Exercise Study (NEEDED), ClinicalTrials.gov identifier: NCT02166216, is a study to see if there is any connection between physical exercise and cardiovascular diseases. Electrocardiogram (ECG) data are gathered in 3 measurements: 1 day before the race, 3 hours after reaching the finish line, and 24 hours after the race. For each measurement the duration of ECG signal is $10\,\mathrm{s}$. The dataset contains above 1000 participants.

3. Method

3.1. Mean Electrical Axis

In this subsection an equation set to find MEA is derived for the Einthoven model. For a vector $\underline{a}$, $\underline{a}^T$ is the transpose. For a matrix $\boldsymbol{A}$, $\boldsymbol{A}^+$, $\mathrm{col}(\boldsymbol{A})$, $\mathrm{row}(\boldsymbol{A})$ are the pseudoinverse, column space of $\boldsymbol{A}$, and row space of $\boldsymbol{A}$, respectively. $\langle\cdot,\cdot\rangle$ denotes the inner product.

To find the MEA in the frontal plane from the limb leads, an orthogonal projection $\hat{\underline{d}}$ of the dipole $\underline{d}$ onto plane $\boldsymbol{W}$ spanned by the lead vectors is first found. Then another orthogonal projection onto the xy (frontal) plane from $\boldsymbol{W}$ is done to find the MEA in the frontal plane. Since the lead vectors only span the plane $\boldsymbol{W}$ an error from the orthogonal complement of $\boldsymbol{W}$ will occur. To minimize this error an extra lead which are not in $\boldsymbol{W}$ must be used to find the dipole $\underline{d}$.

Let $\underline{d} = (d_x, d_y, d_z)^T$ be a dipole vector at time t. The voltage in a lead i at time t is the inner product of $\underline{d}$ with the lead vector $\underline{v}_i = (x_i, y_i, z_i)^T$ is given as

$$V_i = \langle \underline{v}_i, \underline{d}\rangle = \underline{v}_i^T \underline{d}. \tag{1}$$

The lead vector $\underline{v}_i$ is not necessarily a unit vector. From the Einthoven, Burger and Frank triangles the only certainty is the direction of the lead vectors, e.g. see table 1. However, the scaling does not affect the direction of the dipole. Let

$$\boldsymbol{A} = \left[\underline{v}_i, \underline{v}_j\right]^T \tag{2}$$

$$\underline{b} = \left[V_i, V_j\right]^T \tag{3}$$

where $i \neq j$, then a matrix equation of the form

$$\boldsymbol{A}\underline{d} = \underline{b} \tag{4}$$

can be set up. $\boldsymbol{A}$ has rank $r = 2$ and does not have full column rank. Let $\boldsymbol{A} = \boldsymbol{U}\boldsymbol{D}\boldsymbol{V}^T$ be the reduced singular value decomposition [17]. The columns in $\boldsymbol{U}$ form an orthonormal basis set for $\mathrm{col}(\boldsymbol{A})$ and the columns in $\boldsymbol{V}$ form an orthonormal basis set for $\mathrm{row}(\boldsymbol{A})$. $\boldsymbol{D}$ is a diagonal matrix where the entries are non-zero singular values. The pseudoinverse is defined to be

$$\boldsymbol{A}^+ = \boldsymbol{V}\boldsymbol{D}^{-1}\boldsymbol{U}^T. \tag{5}$$

The least square solution of the matrix equation is

$$\hat{\underline{d}} = \boldsymbol{A}^+\underline{b} \tag{6}$$

$$= (\boldsymbol{V}\boldsymbol{D}^{-1}\boldsymbol{U}^T)(\boldsymbol{U}\boldsymbol{D}\boldsymbol{V}^T\underline{d}) \tag{7}$$

$$= \boldsymbol{V}\boldsymbol{V}^T\underline{d}. \tag{8}$$

Hence $\hat{\underline{d}}$ is the orthogonal projection of $\underline{d}$ onto the plane $\boldsymbol{W}$. If the plane $\boldsymbol{W}$ is the frontal plane then it is now possible to find the MEA as an angle

$$\beta = \arctan\left(\frac{\hat{d}_y}{\hat{d}_x}\right), \tag{9}$$

where $\hat{d}_x$ and $\hat{d}_y$ is the x and y component of $\hat{\underline{d}}$. Because of linearity, from eq. (6) it can be seen that the sum of dipole vectors (at different points in time) can be determined from the sum of ECG voltages.

3.2. Net Potential

A method to calculate the MEA is to use a net potential that is the sum of peak voltages of the R and S waves, although it is much simpler to use the max and min value of the QRS complex. Let $x(n)$ be a segment which includes the QRS complex then the net potential can be express as

$$V_{rs} = \max(x(n), 0) + \min(x(n), 0) \tag{10}$$

An alternative to using max and min value of the QRS complex is to use the area under the curve. If the trapezoid method is used then it can be shown that

$$V_{area} = x(0)/2 + \sum_{n=1}^{N-2} x(n) + x(N-1)/2 \tag{11}$$

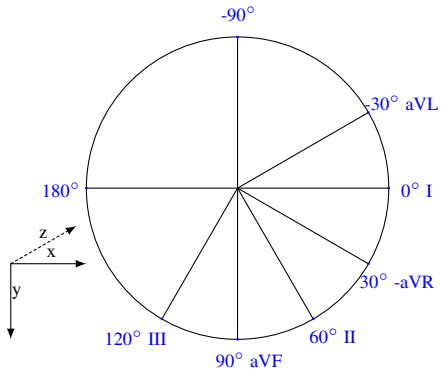

Figure 1. Hexaxial reference system.

where N is the length of the segment $x(n)$. However, from eq. (6) the sum of heart vectors corresponds to the sum of voltages, giving the alternative definition of net potential as

$$V_{sum} = \sum_{n=0}^{N-1} x(n). \tag{12}$$

3.3. Preprocessing

The ECG signals are preprocessed to remove baseline wander, high frequency above 150 Hz [6] and powerline interference. Then signal averaging [18, 19] is performed to find the most common QRS template.

For a successful signal averaging it is necessary to synchronize the signal segments during clustering. The average square difference function (ASDF) method [20] is used to synchronize the segments. The main benefit of this method is perfect synchronization in the absent of noise. The spectrum is also not needed when compared to generalized correlation methods in [21].

Finally an isoelectric (asystole) correction is used to align PQ intervals close to 0 V.

3.4. Assessment

Let a model $X = \{x_{i,j}\} = [\underline{x}_1, \underline{x}_2, \ldots, \underline{x}_N]$ be a matrix of size (K, N) such that each element corresponds to a MEA. Each column denotes an ECG record and each row denotes a lead pair.

Column deviation for column $\underline{x}_j$ is defined to be

$$D_{c,j} = \sqrt{\frac{1}{K}\sum_{i=1}^{K}(x_{i,j} - R_{i,j})^2} \tag{13}$$

where $R_{i,j}$ is a reference MEA, a different model's MEA, or a central value such as the mean, $\mathrm{E}\left\{\underline{x}_j\right\}$. When $R_{i,j} = \mathrm{E}\left\{\underline{x}_j\right\}$ then the column deviation is the standard deviation between lead pairs. Expected column deviation is denoted as

$$M_c = \frac{1}{N}\sum_{j=1}^{N} D_{c,j} \tag{14}$$

The column deviation is a measure of dispersity throughout the lead pair in a record. The expected column deviation is the mean value of the column deviation. A low value indicate a good conformity between central value or reference MEA with the lead pairs' MEA.

Row deviation for a row i is defined to be

$$D_{r,i} = \sqrt{\frac{1}{N}\sum_{j=1}^{N}(x_{i,j} - R_{i,j})^2}. \tag{15}$$

The row deviation is a measure of how much each lead pair differ from a true value $R_{i,j}$.

4. Results

The percentage of ECGs with standard deviation with less than 15° between the lead pairs are 94.2% for V_{rs}, 98.7% for V_{area} and 98.7% for V_{sum}. Table 2 shows the lead pair deviation (row deviation). Table 3 shows the expected column deviation. Table 4 shows axis deviation according to axis deviation criteria defined in [7]. Not a number (NaN) is added to the undefined category.

Table 2. Row deviation. (Degree, °)

		Vrs		Varea		Vsum	
$R_{i,j}$		mean	ref.	mean	ref.	mean	ref.
I	II	7.0	14.4	5.0	10.2	5.0	10.2
	III	6.5	13.7	7.0	12.3	7.1	12.3
	aVF	4.8	13.1	4.6	12.0	4.6	12.0
	aVL	10.3	15.2	10.0	11.3	10.1	11.3
	aVR	10.7	14.7	8.9	9.6	9.0	9.7
II	III	5.2	13.5	4.7	10.0	4.8	10.1
	aVF	7.2	14.5	7.4	12.6	7.5	12.7
	aVL	5.0	13.1	4.2	9.9	4.2	9.9
	aVR	8.9	15.6	7.4	11.6	7.5	11.6
III	aVF	9.3	15.0	7.5	13.5	7.5	13.6
	aVL	10.6	15.4	6.9	12.3	6.9	12.3
	aVR	5.0	13.5	5.3	12.4	5.3	12.4
aVR	aVF	5.9	13.9	5.4	12.2	5.5	12.3
	aVL	8.6	12.7	9.1	10.5	9.2	10.5
aVF	aVL	5.9	13.1	4.5	11.9	4.6	11.9

5. Discussion and Conclusion

The results show that the two net potentials V_{area} and V_{sum} are quite similar. There is a slight difference which is shown in table 3. The last column shows deviation between V_{area} and V_{sum} to be 0.08°. The deviation between

Table 3. Expected column deviation. (Degree, °)

$R_{i,j}$	mean	ref.	Varea
Varea	3.26	4.53	0.00
Vrs	5.81	9.75	9.59
Vsum	3.29	4.56	0.08

Table 4. Number of axis deviation from mean lead pairs.

	Vrs	Varea	Vsum	ref.
Left axis deviation	60	113	113	107
Right axis deviation	43	79	79	59
Normal	2908	2815	2815	2841
Undefined	1	5	5	11

V_{area} and V_{rs} is $9.52°$. Table 3 also shows that V_{area} has the smallest deviation between lead pairs and with respect to the reference MEA. Table 2 shows that lead pair I-aVL, I-aVR, III-aVL, and aVR-aVL are amongst lead pairs which differ most from the mean. Omitting these might give less variability. Table 4 shows that different methods give rise to different axis deviation. Because of the disparity between the models a standard is highly sought otherwise it might lead to erroneous clinical conclusions. The percentage of ECGs with less than $15\,\%$ standard deviation between the leads is higher for V_{area} and V_{sum} ($98.7\,\%$) compared to V_{rs} ($94.2\,\%$). V_{area} and V_{sum} also give less variability between the leads therefore clinical fidelity should be based on them.

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