A Study on the Characteristics Influencing the Pressure in the Inlet of a One-Dimensional Model of Arterial Structure

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Abstract

Blood pressure carries crucial information about the response of the arterial system to the beating heart. Extracting useful information from the blood pressure plays a significant role in diagnosis and treatment of cardiovascular disease such as hypertension.
There are many studies focusing on the existence of reflection waves in the ascending aorta and its influence on the amplitude of pressure. However, there is an ongoing debate about the origin and distance that a reflection wave can travel.
In this study, an one-dimensional (1D) model of a series of bifurcation is simulated in order to analyse the effect of bifurcations on the pressure amplitude. A comparison has been made between the pressure in the inlet of the model and the pressure in the terminal ends. The results show an exponential decay with increasing numbers of bifurcations.

1. Introduction

Blood pressure waves generated by heart contraction encounter many bifurcations and changes in the structure of arterial tree, creating backward waves travelling towards the heart. [1] The existence of backwards waves, their magnitude, and their effect on increasing the pressure measured in aortic root has been recently revisited with an ongoing debate, particularly for the outstanding question about the origin of the reflected wave present in the aortic root, and how far the reflected waves could travel in the arterial tree. [2]
Haemodynamical simulations using computational models can be applied to assess different vessels in the systemic circulation that might not be accessible in human, or in certain conditions cannot be achieved *in-vivo*. 1D modelling has the ability to capture the global dynamics of the arterial system, and it has been validated against *in-vitro* [3] and *in-vivo* experiments [4, 5]. The aim of this study is to investigate how the generation of bifurcations affect the magnitude of reflected waves, and how far reflected waves can travel in the backward direction with a discernible magnitude. We illustrate this by considering a simple cases of in-series connection bifurcations ended by total absorption in order to limit the reflections originating from the bifurcations only.

2. Methods

The non-linear one-dimensional (1D) equations of blood flow in elastic tubes have been used to develop a computational model. The equations consider blood as an incompressible and Newtonian fluid. This model is used to trace the waves as they travel a branching system as a surrogate of the arterial system. For each segment in the arterial system, the conservation of mass and momentum is applied and they can be written respectively:

$$\frac{\partial A}{\partial t} + \frac{\partial (AU)}{\partial x} x = 0 \tag{1}$$

$$\frac{\partial U}{\partial t} + U\frac{\partial U}{\partial t} = \frac{-1}{\rho}\frac{\partial P}{\partial x} + \frac{f}{\rho A} A \tag{2}$$

where x is the axial coordinate along the segments, t is time, $U(x,t)$ is the mean blood axial velocity, $A(x,t)$ is the cross-sectional area of each segment, $P(x,t)$ is the internal pressure in each cross-section, and $\rho = 1050Kgm^{-3}$ is the density of the blood.
$f = -2(\gamma + 2)\mu\pi U$ is the friction force per unit length. γ is determined by the assumed velocity profile and it is a non-dimensional correction factor. Based on findings of [6] , $\gamma = 9$ is a good agreement with the values of the experimental data and considers the velocity profile to be flat with no-slip condition.The viscosity of of blood (μ) is 4 mPa s. Equations (1) and (2) consist of three unknowns namely pressure, velocity, and area.Therefore, the pressure-area relationship is coupled to the governing equations to close the system.

$$P(x,t) = P_0 + \beta(\sqrt{A(x,t)} - \sqrt{A_0}) \tag{3}$$

Where β is

$$\beta(x) = \frac{4}{3}\sqrt{\pi} h E \quad (4)$$

with $A_0(x)$ as area in initial condition where $(P, U) = (0, 0)$,$h(x)$ as wall thickness, and $E(x)$ as Young's modulus. This law assumes the arterial wall to be thin, elastic, homogeneous, and incompressible.The speed that the waves travel in tubes also known as pulse wave velocity (c) is calculated with Moens-Korteweg equation:

$$c(x) = \sqrt{\frac{\beta(x)}{2\rho}} A^{\frac{1}{4}}(x) \quad (5)$$

By inserting any perturbation into a vessel, a wave will propagate with the speed of $U + c$ in forward direction and $U - c$ in the backward direction. The resulted changes in pressure and velocity in arterial tree can be separated into forward-travelling and backward-travelling components, where forward direction is away from the heart to the periphery and the backward direction is waveform travelling from periphery towards the heart. The changes in total pressure and velocity are sum of changes in their forward and backward components $dP = dP_f + dP_b$ and $dU = dU_f + dU_b$. By using the mentioned equations and the water-hammer equation,

$$dP_f = \rho c \, dU_f, \quad dP_b = -\rho c \, dU_b$$

the forward and backward components of pressure and velocity are:

$$dP_{f,b} = \frac{1}{2}(dP \pm \rho c \, dU), \quad dU_{f,b} = \frac{1}{2}(dU \pm \frac{dP}{\rho c}). \quad (6)$$

Wave intensity analysis (WIA) is flux of energy carried by the wave per cross-sectional area of segments and it is shown with dI. Positive value for WIA $(dI > 0)$ shows forward travelling waves, and the negative value $(dI < 0)$ shows the reflections. WIA is the product of changes of pressure and velocity during a small time interval.WIA has SI unit of W/m^2 and provides an insight if the direction and timing of the waves. Using equations 6 and Water-hammer equation, dI can be calculated by:

$$dI_{f,b} = dP_{f,b} \, dU_{f,b} = \frac{\pm 1}{4\rho c}(dP \pm \rho c \, dU)^2 \quad (7)$$

Since the length of the segments (l) are pre-defined and the Pulse wave velocity can be calculated with predefined material properties, the transient time can be calculated by the simple formulation of velocity and distance:

$$t = \frac{l}{c} \quad (8)$$

Many studies has shown evidence that the arterial bifurcations are well-matched for forward waves, which means the backward waves are reflected more. Based on the measurements of human bifurcations in different site of arterial system, the area ration between the daughters and the parent vessels was 1.18 ± 0.04 .[7] Therefore, in this study the relation between the parent segment and the daughter tubes is $\alpha = 1.15$, where

$$\alpha = \frac{(A_1 + A_2)}{A_0} \quad (9)$$

The mother tube is referred as 0 and the daughters are 1 and 2. For detailed explanation of the numerical modelling we refer the reader to [8]. On the basis of using the above-mentioned equations, two sets of 1D simulations have been carried out.The length of all the segments are 5-metres in order to prevent the overlapping of the reflections.

2.1. Five-consecutive bifurcations

In the first experiment, a Gaussian-shaped pulse is inserted from the inlet of the mother tube followed by 5 generations of bifurcations (figure 1). With this model the changes of pressure in the inlet is monitored while there is a total blockage in the 5th bifurcation. The boundary condition for the other terminals is complete absorption.

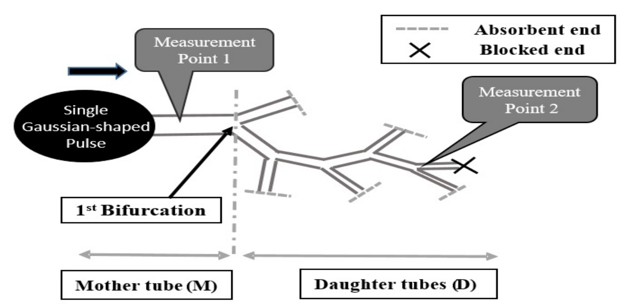

Figure 1. Schematic presentation of the 5 consecutive bifurcation structure with the Gaussian-shaped pulse inserted in the inlet. Each segment is 5 metres long.

2.2. 15 consecutive bifurcations

A structure of 15 consecutive bifurcation is simulated in 1D model. The changes of pressure in the beginning of the mother-tube (M) is analysed in 15 different simulations (figure 2).In each simulation, a Gaussian-shaped impulse is inserted to one of the terminal point of the daughter tubes, and then the pressure measured in the inlet of the mother tube is compared with the amplitude of input pressure from the peripheral points. The boundary condition of the other daughter tubes and the inlet of the mother tube were kept

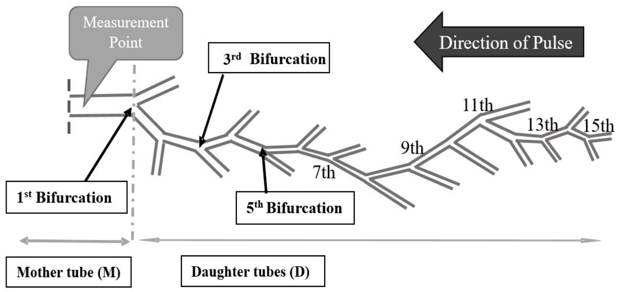

Figure 2. Schematic presentation of the 15 consecutive bifurcation structure. Each segment is 5 metre long.

absorbent, in order to investigate the magnitude of the arrival waves from the specific terminal point to the mother tube.

3. Results

3.1. Five consecutive bifurcation

Since the length and the pulse wave velocity of the segments are known, the transient time of the reflections can be calculated theoretically using equation 8. Figure 3 shows pressure and velocity changes in measurement point 1. Each wave is named and and further analysed in table 1. The reflection originating from the blocked terminal end in 5th bifurcation is arriving to the inlet of mother tube at 7.9 seconds. As shown in figure 3 , at 7.9 second the existing amplitude of pressure is negligible and overlapped with other artefact which cannot be originated their source. WIA has be used for the measurements in the inlet of the structure and in the blocked segment (figure 4 and 5). There is no evidence of the reflection from the blocked end in the mother tube although there is a reflection travelling backward from the blocked end.

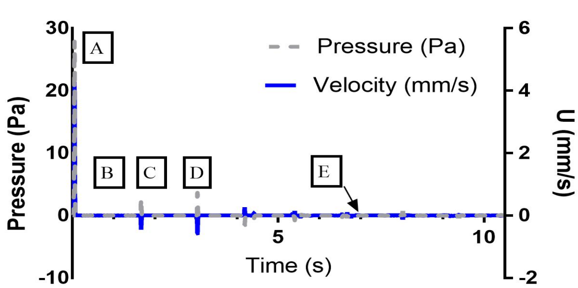

Figure 3. Pressure and velocity variation in the mother tube of the five generation bifurcations. Each label in the figure corresponds to a wave. More detail about the waves is presented in table 1.

Table 1.Waves presented in figure 3

Measurement Point 1			
Wave	Path	Arrival Time	Pressure
A	Input	0	27
B	1,-1	1.62	2.7
C	1,2,-2,-1	2.99	3.742
D	1,2,3,-1,-2,-3	4.18	-1.55
E	1,2,3,4,5,-5,-4,-3,-2,-1	7.979	?

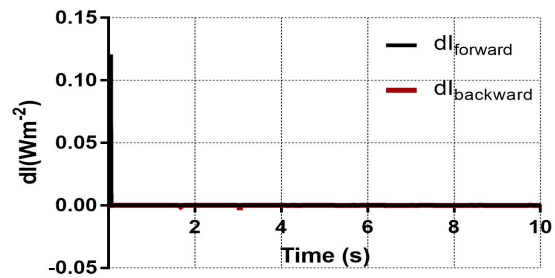

Figure 4. Wave Intensity Analysis in the first measurement point. The reflection from the first and second bifurcation can be seen at 1.6s and 3s, respectively.

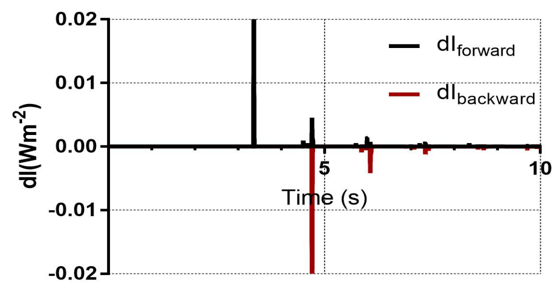

Figure 5. Wave Intensity Analysis in the measurement point 2 in the blocked segment.

3.2. 15 consecutive bifurcation

Since the previous simulations were challenging to interpret and quantify the amplitude of the pressure, in these sets of simulations we imposed an ”artificial reflection” considerable enough to be detected in the inlet of the mother tube. The pressure measured in the mother tube P(M) is compared against the pressure pulse that was inserted in terminal points of each daughter tubes P(D), and the results are presented as a ratio (Rp) of the measured pressures (figure 6).

4. Discussion

Compared to the existing theoretical models of arterial system in literature, the models studied here are relatively

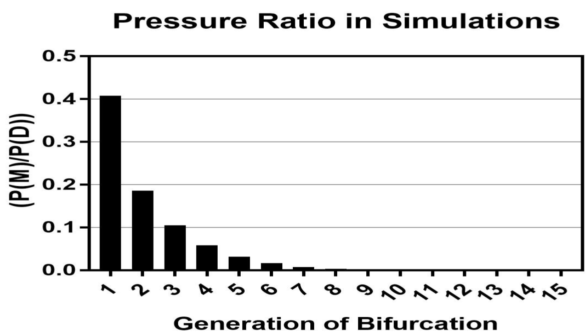

Figure 6. Ratio of the measured pressure in the mother tube P(M) over the pressure measured in the daughter tubes P(D) in 15 generation of bifurcations

simple. However, it reproduces the features of reflected waves in a branching system similar to the arterial tree by easy-to-grasp systems. The absence of reflected waves generated from other terminal points might be an oversimplification, but it allowed the tracking of changes in pressure waveform passing different bifurcations . In addition, although not physiological, the length of 5 metres of each segment was used in order to prevent the over-lapping of waves. In contrast to the arterial system with multiple sites of reflection from tapering and peripheral resistance, the models in this study focus on effect of the reflections solely because of bifurcations. Based on the results of the first experiment, reflected waveforms are discernible when originating from the two initial bifurcations. As the wave travels throughout the model, the magnitude of the reflection decreases and even the reflection from the blocked terminal point is too small to be discerned without magnification of the time period 10-15s.
While in the second part of this study, the reflection waves from the terminal points were considerable enough to be observed in the inlet of the structure even after travelling through 15 reflection sites. The results of these simulations provided an overview of the ratio of reflections originating from the end point of different generation of bifurcations. This may indicate that more than half of the waves are re-reflected in the arterial tree.

5. Conclusion

Similar to many branches of applied mathematics, it is necessary to analyse simple models before increasing the complexity of the modifying feature present in reality. Therefore, we started by considering a structure of bifurcations without peripheral reflections and 5-metre segment and analysing the propagation of pressure throughout the bifurcations. A significant decrease in the amplitude of the reflected wave reaching to the root of bifurcations is evident in both experiments. With the increase in number of bifurcations, as is the case *in-vivo*, single reflections originating at the periphery may not be discernible at the aortic root. Further work is required to examine the decay of the reflected waves in a larger number of segments and bifurcations similar to those of the arterial system. In addition, Introducing the effect of tapering to the bifurcations can give more insight about propagation of the pressure in the arterial system. The results of the computational simulations can be validated with *in-vitro* setup.

Acknowledgements

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