Finite Element Analysis of Blood Flow via a Shunt from the Left Ventricle to the Distal Segment of a Stenosed Coronary Artery

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Abstract

New methods are currently being developed that provide a direct connection between the left ventricle and the distal portion of an obstructed coronary artery. Optimal design of such a device can be aided by the use of numerical analysis. Here, computational results are presented showing the hemodynamic flows through a bypass shunt providing flow to a stenosed coronary artery. A PISO type finite element method is used to solve the three-dimensional incompressible Navier-Stokes equations for flow passing through a bypass shunt into the occluded coronary artery. Time-varying inlet flow from the left ventricle is obtained from a simulation of the entire coronary and systemic circulations. The main objective of this study is to delineate the influence of shunt angle on detailed flow patterns. Three different shunt angles are examined. Computational results show a recirculating region generated near the junction of the coronary artery with the bypass shunt. Secondary flow is induced in the cross-sectional plane perpendicular to the axis of the artery and is progressively attenuated downstream. Among the three cases studied, secondary flows are greatest with the 90° shunt angle. The maximum pressure drop between inlet to outlet also increases with increasing shunt angle.

1. Introduction

Two surgical methods are commonly used to treat coronary heart disease: aortic-coronary bypass surgery and coronary angioplasty. Despite the widespread success of these procedures, many problems still exist requiring an alternative form of treatment. Many patients, for example, continue to experience severe angina even after these procedures; in others, the vessels become occluded rapidly, requiring further intervention. In some of these cases neither surgery nor angioplasty is possible and alternative solutions need to be considered. One such alternative is the placement of a stented shunt (termed VSTENT^{TM}, Percardia, Inc.) that directly connects the left ventricle to the distal segment of the obstructed coronary artery [2]. Since this dramatically alters the nature of blood flow through the vessel, and also alters local hemodynamics, numerical simulation has been employed to investigate the resultant flows and to help optimize the design and placement of the devices.

In this study we present the computational simulation of blood flow through the bypass shunt. A lumped parameter model is used to prescribe the flow boundary condition at the inlet of the shunt while a PISO type finite element model is used to study the detailed flow patterns within the shunt and adjacent artery and to identify regions of potential flow separation and high or low shear stress. Shunt angle is systematically varied to examine its effect on local flow conditions.

2. Numerical method

A PISO type finite element method is used to solve the incompressible Navier-Stokes equations:

$$\nabla \cdot \tilde{u} = 0 \tag{1}$$

$$\rho \frac{\partial \tilde{u}}{\partial t} + \rho \tilde{u} \cdot \nabla p = -\nabla p + \mu \nabla^2 \tilde{u} \tag{2}$$

Here, we use the notation as follows: $\tilde{u}$, the velocity vector; p, the pressure; ρ, the fluid density ; and μ, the dynamic viscosity coefficient. By applying Galerkin finite-element discretization for Eqs. (1) and (2), we can obtain a compact matrix form

$$MU' + K(U)U = CP + F \tag{3}$$

$$C^T U = 0 \tag{4}$$

Here, U is the global velocity vector containing nodal vales of u, v, and w; P is a global pressure vector, and U' is the time derivative of U; M is the mass matrix, and $K(U) = A(U) + D$ is the matrix consisting of convection terms plus diffusion terms; C is the gradient matrix, and C^T is the divergence matrix; F is a force vector, which incorporates the natural boundary conditions. Application of the implicit Euler method to the differential matrix Eqs. (3) and (4) leads to

$$(M/\Delta t + K)U^{n+1} = MU^{n}/\Delta t + CP^{n+1} + F^{n} \quad (5)$$

$$C^{T}U^{n+1} = 0 \quad (6)$$

where the convection terms are locally linearized.

The present numerical method consists of a predictor step and two corrector steps at each time step. In the predictor step, the pressure field prevailing at the previous time level is initially used in Eq. (5) to obtain the velocity components; the matrix equation is solved by the iterative method PGGS [3].

In the first corrector stage, the corrected velocity and its corresponding predicted pressure are determined simultaneously so that the continuity condition (4) is satisfied. Combining the velocity correction equations with the continuity equation, we obtain the pressure increment equation. To get the pressure field from the pressure increment equation, we used the iterative matrix solver ICCG [4]. The pressure and velocity are finally obtained in the second corrector step, the procedure of which is similar to the first corrector step. More information on the present numerical algorithm can be obtained from Shim and Chang [5].

3. Results and discussions

The proposed TMR procedure introduces a direct connection between the left ventricle and the distal segment of the obstructed artery. For this purpose, a shunt connecting the left ventricle to the coronary artery is inserted in the heart muscle. The shunt shape has some taper to utilize the benefit of asymmetric flow resistance. The schematic diagram of the procedure is represented in Figure 1.

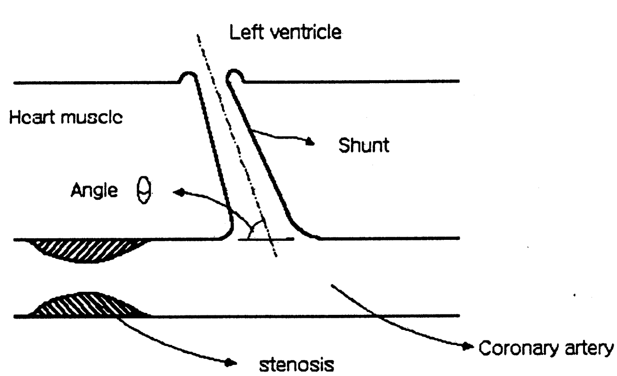

Figure 1. Schematic of the proposed surgical procedure.

The computational model in this study is depicted in Figure 2 showing the bypass shunt connecting the occluded coronary artery with the left ventricle. An unstructured tetrahedral mesh is used having four velocity and four pressure nodes.

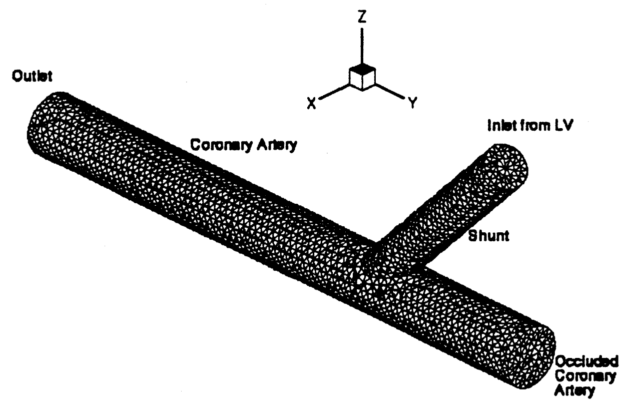

Figure 2. Schematic of the computational model and the surface mesh at 45° shunt angle. The total number of nodes and elements used in this model are 13923 and 65173, respectively.

To delineate the hemodynamic effect of stent replacement we implemented a computational code that can simulate the system dynamics of the cardiovascular system after procedure. We inserted the coronary circulation into an existing model of the entire cardiovascular system developed by Heldt et al. [6]. The coronary circulation consists of three compartments: the coronary arteries, the coronary capillaries and the coronary veins. Application of Kirchhoff's law to each node in the lumped parameter hemodynamic model leads to a matrix equation of the form:

$$d\mathbf{p}/dt = A\mathbf{p} + \mathbf{b} \quad (7)$$

Here $\mathbf{p}$ is the vector of compartmental pressures, A represents the time constants for exchange between compartments, and $\mathbf{b}$ is the input to the system. This initial value problem of ordinary differential equation is solved using fourth order Runge-Kutta method.

In the present computation, flow rate is prescribed at the inlet originating from left ventricle, which is calculated from the lumped parameter model (Figure 3). Here, Q_{coa} is flow rate to the coronary capillaries and Q_{sh} is flow rate through the bypass shunt. The inlet velocity boundary condition is specified using the time-varying shunt flow rate Q_{sh}.

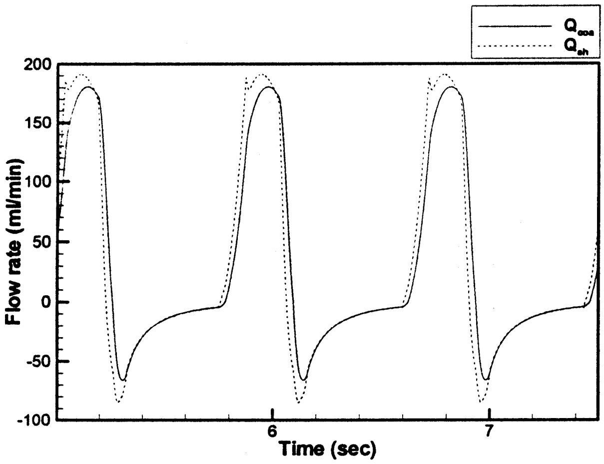

Figure 3. The shunt flow rate, Q_{sh}, used as the inlet boundary condition as a function of time.

Figure 4 depicts the velocity vectors at the symmetric cutting plane as a function of the shunt angle at the time of maximum inlet flow rate. A large recirculating region can be seen near the junction, the strength of which increases with increasing shunt angle.

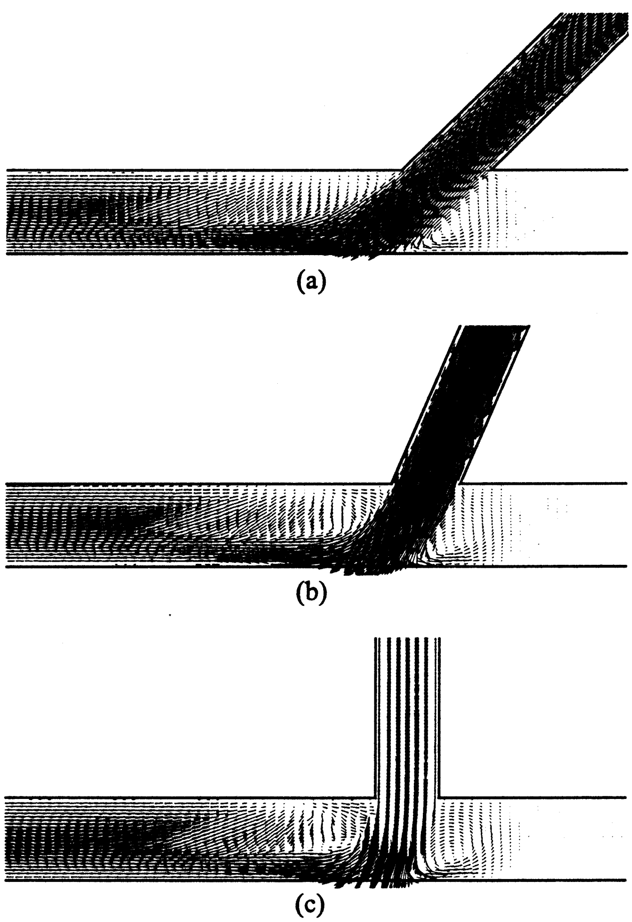

Figure 4. Velocity vectors at the symmetric plane X=0 as a function of the shunt angle. (a) θ=45°, (b) θ=67°, (c) θ=90°.

The velocity vectors in the cross-sectional planes at the proximal and at the distal part of the occluded coronary artery are shown in Figure 5, showing significant secondary flow. This indicates the formation of two counter-rotating vortices in the artery. This secondary flow is attenuated as it moves into downstream.

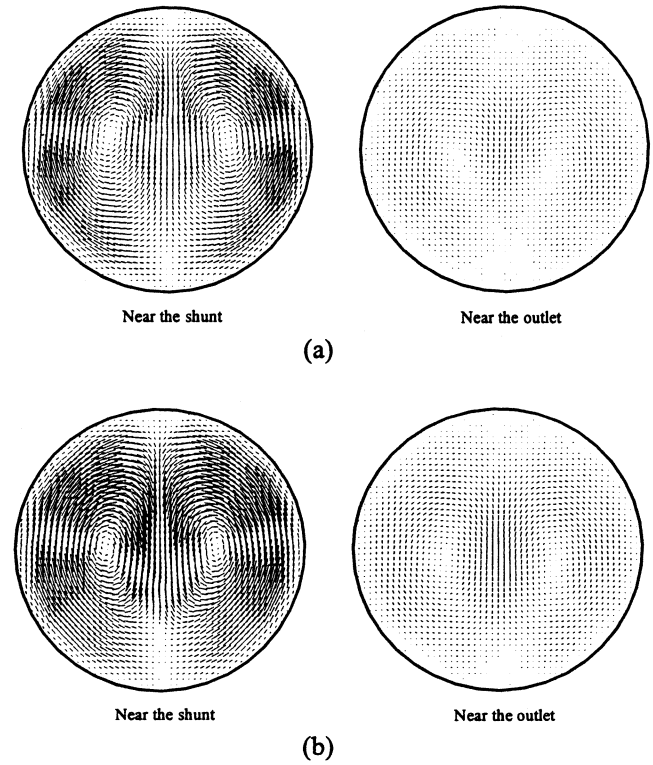

Figure 5. Velocity vectors at the cutting plane of constant Y as a function of the shunt angle. (a) θ=45°, (b) θ=90°.

The maximum pressure drop between inlet and outlet during one cardiac cycle (Figure 6) was found to increase with increasing shunt angle. As shown in Fig. 4, the recirculating flow exists near the junction of the bypass shunt with artery. Its strength increases according to shunt angle, indicating the higher energy loss in case of higher turning angle.

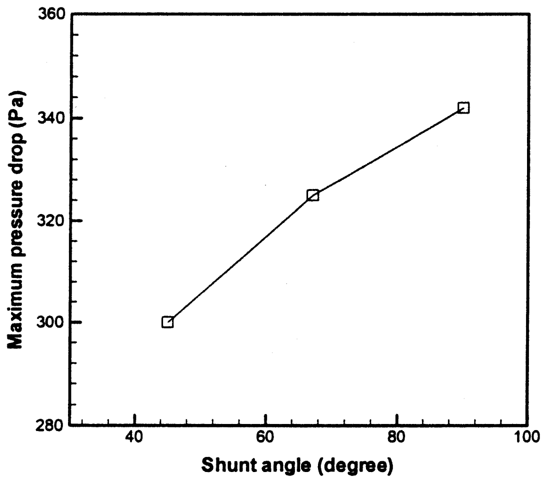

Figure 6. Maximum pressure drop over a cardiac cycle as a function of the shunt angle.

4. Conclusions

In the simulations presented, we demonstrate a method by which the design of the TMR shunt can be optimized with regard to dimensions, orientation entering the vessel, and geometric shape. Here we vary just one of these, the entry angle, and demonstrate significant variations in the flow pattern in the native vessel.. Simulations of this type are critical in that they provide the designer with insights into how undesired flow characteristics might be avoided. In this particular example, we focused on flow separation, which is especially detrimental due to their tendency to promote the formation of blood clots or thrombi. In general, the surgeon strives for a situation in which the shear stress is relatively uniform, recirculation is absent, and regions of low or oscillatory shear stress are minimized. A design that has been optimized from a fluid dynamic perspective would conceivably reduce the tendency for subsequent occlusion and increase the potential for successful outcome.

Novel aspects of the present simulation are the use of the new PISO type finite element method, and the coupling of the finite element solution to a lumped parameter representation of the rest of the circulation. By means of this coupled approach, a detailed solution can be obtained where the interest lies in the particular flow patterns while the more global response to the local changes can also be monitored. Moreover, the global solution provides the flow boundary condition for the local simulation, and adjusts to changing conditions locally, or to a variety of simulated hemodynamic states of the patient.

Acknowledgements

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