

Estimation of Spontaneous Respiratory Rate from Photoplethysmography by Cross Time-Frequency Analysis

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Abstract

In this paper, a methodology to indirectly estimate the respiratory rate from the photoplethysmography (PPG) signal is presented. The possibility to reliably estimate respiratory rate from the PPG signal is particularly appealing since PPG is simple, comfortable and cheap. The underlying hypothesis of this methodology is that respiration provokes simultaneous changes in the pulse interval, amplitude and width of the PPG signal. These respiratory related changes are combined together by cross time-frequency analysis, performed by smoothed pseudo Wigner-Ville distribution, in order to obtain indirect estimates of respiratory rate. The algorithm is designed to yield estimates only when the estimation is robust. In 17 spontaneous breathing subjects, among which 7 were characterized by a respiratory rate lower than 0.15 Hz, this methodology provided accurate estimates, being the median error 0.00 ± 1.95 mHz ($0.00 \pm 0.75\%$) and the interquartile range error 7.81 ± 6.10 mHz ($3.00 \pm 4.03\%$).

1. Introduction

Respiratory activity is reflected in many cardiovascular signals, such as the heart rate and the arterial pressure, mainly due to intrathoracic pressure changes and autonomic nervous modulation. The presence in cardiovascular variability of oscillations synchronous with respiration, makes it possible to indirectly estimate respiratory rate from cardiovascular signals. The use of indirect estimates of the respiratory rate from cardiovascular signals is relevant since in many situations the respiratory rate cannot be directly measured. Different methodologies for the extraction of the respiratory rate from the ECG have been proposed in the literature [1]. Photoplethysmography (PPG) signal has been applied in many different clinical settings [2], including the monitor-

ing of blood oxygen saturation, heart rate [3], blood pressure, cardiac output and respiration [4]. Given its simplicity, low-cost and that it is widely used in the clinical routine, it is desirable to maximize the PPG potential by exploring additional measurements that can be derived from it. It is worth noting that oximetry systems can provide multiple information using only one sensor, making its use simpler, more comfortable and cheaper than multiple sensor devices.

In this paper, we presented a methodology to indirectly estimate the respiratory rate from the PPG signal. The underlying hypothesis of this methodology is that respiration provokes simultaneous changes in the pulse interval, amplitude and width of the PPG pulses. These respiratory related changes are combined together by cross time-frequency (TF) analysis, performed by smoothed pseudo Wigner-Ville distribution (SPWVD).

The methodology has been specially designed to provide robust estimates. To this end, coherence analysis is used with a twofold objective: to perform a sort of control of the accuracy of the estimates and to localize signal-dependent TF regions in which respiratory rate is extracted.

The proposed methodology is assessed in 17 spontaneously breathing subjects undergoing a tilt table test.

2. Methods

As shown in the block diagram of Fig. 1, the methodology applies to the variability of given features of the PPG signals, $x_i(t)$, which are affected by respiration. In this study, $x_i(t)$ are the pulse interval variability (PIV), pulse amplitude variability (PAV) and pulse width variability (PWV). Once that $x_i(t)$ have been estimated, the algorithm is composed of the following main parts:

(i) Estimation of the auto and cross TF spectra, $S_k^{ij}(t, f)$, and coherence, $\gamma_k^{ij}(t, f)$, between signals $\{x_i(t), x_j(t)\}_k$, with $(i, j) \in \{1, \dots, N\}$ and $k \in \{1, \dots, (N-1)N/2\}$, where N is the total number of signals, and k is the index numbering the cross TF spectra and coherence.

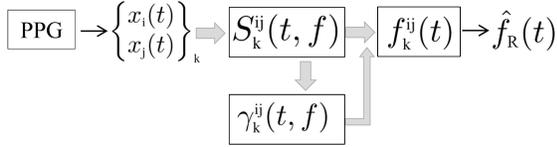


Figure 1. Block diagram of the algorithm. $x_i(t)$ and $x_j(t)$ represent signals derived from the PPG signal which are affected by respiration. Here $\{x_i(t), x_j(t)\} \in \{PIV, PAV, PWV\}$.

- (ii) Estimation of the instantaneous frequency, $f_k^{ij}(t)$, with $i \neq j$, of the respiration-related component of $S_k^{ij}(t, f)$.
- (iii) Combination of $f_k^{ij}(t)$ to obtain estimates of the respiratory frequency $\hat{f}_R(t)$.

2.1. Cross time-frequency analysis

The smoothed pseudo Wigner-Ville distribution (SPWVD) was used to estimate TF spectra and coherence functions. The TF spectra between $\{x_i(t), x_j(t)\}_k$, $S_k^{ij}(t, f)$, were obtained by taking the Fourier transform of the product between the ambiguity function $A_k^{ij}(\tau, \nu)$ and an elliptical exponential kernel $\Phi(\tau, \nu)$:

$$S_k^{ij}(t, f) = \iint_{-\infty}^{+\infty} \Phi(\tau, \nu) A_k^{ij}(\tau, \nu) e^{j2\pi(\nu t - \tau f)} d\nu d\tau \quad (1)$$

$$A_k^{ij}(\tau, \nu) = \int_{-\infty}^{\infty} x_i\left(t + \frac{\tau}{2}\right) x_j^*\left(t - \frac{\tau}{2}\right) e^{-j2\pi\nu t} dt \quad (2)$$

$$\Phi(\tau, \nu) = \exp\left\{-\pi\left[\left(\frac{\nu}{\nu_0}\right)^2 + \left(\frac{\tau}{\tau_0}\right)^2\right]^{2\lambda}\right\} \quad (3)$$

The iso-contours of $\Phi(\tau, \nu)$ are ellipses whose eccentricity depends on parameters ν_0 and τ_0 [3]. Parameters ν_0 and τ_0 are used to change the length of the ellipse axes aligned along ν (i.e. the degree of time filtering) and τ (i.e. the degree of frequency filtering), respectively. The parameter λ sets the roll off of the filter.

Time-frequency coherence, which measures the degree of local coupling between two signals, is also estimated by SPWVD. To estimate the TF coherence, the filtering provided by $\Phi(\tau, \nu)$ should completely suppress the interference terms, since they may cause coherence estimates to take values outside the range $[0, 1]$, thus losing their physical interpretation. As long as the degree of TF filtering is strong enough, TF coherence by SPWVD is obtained as:

$$\gamma_k^{ij}(t, f) = \frac{|S_k^{ij}(t, f)|}{\sqrt{S_k^{ii}(t, f)S_k^{jj}(t, f)}} \quad (4)$$

The TF regions where the local coupling is significant are localized by a hypothesis test. The test is based

on the comparison of $\gamma_k^{ij}(t, f)$ with a threshold function $\gamma_{TH}(t, f; \alpha)$, obtained as the $(1 - \alpha)^{th}$ percentile of the statistical distribution $\Gamma(t, f) = \{\gamma_1^{ww}(t, f), \dots, \gamma_k^{ww}(t, f), \dots\}$, where $\gamma_k^{ww}(t, f)$ is the TF coherence between the k^{th} realization of two white Gaussian noises. The significance level α represents the probability of wrongly detecting local coupling between two signals. Thus, the lower α , the higher $\gamma_{TH}(t, f; \alpha)$.

2.2. Estimation of respiratory frequency

Respiratory frequency is estimated in two steps: first, the instantaneous frequencies of the respiration-related spectral component are estimated from the cross TF spectra; second, these estimates are combined together.

For every couple of signals $\{x_i(t), x_j(t)\}_k$, the instantaneous frequency of the respiration-related components is estimated in a signal-dependent region Ω_{ijk}^α of the cross TF spectra, whose infimum, f_M , is estimated as follows:

(i) Estimate $\overline{\gamma(f)} = \prod_k \overline{\gamma_k^{ij}(f)}$, where $\overline{\gamma_k^{ij}(f)}$ is the temporal mean of $\gamma_k^{ij}(t, f)$.

(ii) If $\overline{\gamma(f)}$ is characterized by two spectral peaks, f_M is estimated as the frequency which corresponds to the minimum in between the 2 spectral peaks.

(iii) If $\overline{\gamma(f)}$ has only one spectral peak, $f_M = 0.05$ Hz.

The region Ω_{ijk}^α is defined as that portion of the TF domain in which $f \in [f_M, 0.5]$ Hz and the coherence is significant:

$$\Omega_{ijk}^\alpha = \{(t, f) \in (\mathbb{R}^+, [f_M, 0.5\text{Hz}]) \mid \gamma_k^{ij}(t, f) > \gamma_{TH}(t, f; \alpha)\} \quad (5)$$

For every couple of signals $\{x_i(t), x_j(t)\}_k$, the instantaneous frequency $f_k^{ij}(t)$, with $k \in \{1, \dots, (N-1)N/2\}$, is estimated as:

(i) Estimate the global maxima of $S_k^{ij}(t, f)$, with $(t, f) \in \Omega_{ijk}^\alpha$. These maxima, $m(t)$, are used as preliminary respiratory estimates.

(ii) Localize intervals T_m , during which an abrupt change, $\frac{d}{dt}m(t) > \pm\Delta f$, followed in less than Δt by another abrupt change of opposite sign, $\frac{d}{dt}m(t) > \mp\Delta f$, occur. Here $\Delta f = 0.04$ Hz and $\Delta t = 10$ s.

(iii) $\forall t \in T_m$, consider all the local maxima, or inflection points, of $S_k^{ij}(t, f)$ inside Ω_{ijk}^α , whose frequencies are called f_s . The f_s which minimizes $|f_s - f^m|$ is called $f_{s,m}$, where f^m is the median value of $m(t)$, estimated in a 2 min temporal window centered in t .

(iv) The instantaneous frequency of the respiration-related component from the k cross TF spectrum is estimated as:

$$f_k^{ij}(t) = \begin{cases} f_{s,m} & \text{if } |f_{s,m} - f^m| < \Delta f \\ m(t) & \text{if } |f_{s,m} - f^m| \in [\Delta f, 2\Delta f] \\ \emptyset & \text{if } |f_{s,m} - f^m| > 2\Delta f \end{cases} \quad (6)$$

where \emptyset stands for empty set.

The estimated respiratory frequency is the median of $f_k^{ij}(t)$:

$$\hat{f}_R(t) = \text{median}_{k \in [1, (N-1)N/2]} f_k^{ij}(t) \quad (7)$$

3. Materials

Seventeen healthy subjects (age 28.5 ± 2.8 years, 11 males) underwent a tilt table test with the following protocol: 4 min in early supine position, 5 min head-up tilted to an angle of 70° and 4 min back to later supine position [3]. The automatic bed took about 18 s to move from 0° to 70° . No subject had cardiorespiratory pathologies. Among the spontaneous breathing subjects, 7 breathed at a frequency rate $f_R(t) < 0.15$ Hz for at least one min, while 5 during the entire test.

The PPG signal was recorded from index finger using the Biopac's PPG100C amplifier with the TSD200 transducer with a sampling frequency of 250 Hz, whereas standard lead V4 ECG signal was recorded using the Biopac's ECG100C amplifier with a sampling frequency of 1 KHz. The respiratory signal was recorded through a strain gauge transducer with a sampling frequency of 125 Hz.

The pulses in the PPG signal were detected by following the procedure described in [3]. Briefly, the PPG signal was resampled at 1 KHz, and the n^{th} pulse was localized as the maximum in an interval going from 150 ms after the n^{th} QRS to the $(n+1)^{\text{th}}$ QRS in the ECG signal. A PPG artifact detector was also applied to suppress pulses from PPG corresponding to artefacts [3]. From the temporal location of the n^{th} pulse wave, t_{p_n} , the pulse interval signal was obtained by interpolating at 4 Hz with 5th order splines the series $(t_{p_n} - t_{p_{n-1}})$. The effect of abnormal beats in the pulse interval was corrected by applying a methodology based on the integral pulse frequency modulation model, and the pulse interval variability (PIV) signal was obtained by high pass filtering with a cut-off frequency of 0.03 Hz. The pulse amplitude variability (PAV) signal was obtained by first interpolating at 4 Hz the series $x_{\text{PPG}}(t_{p_n})$, where $x_{\text{PPG}}(t)$ represents the resampled PPG signal, and by subsequently high pass filtering with a cut-off frequency of 0.03 Hz. The pulse width variability (PWV) was obtained from $x_{\text{PPG}}(t)$ by following the procedure describe in [5].

To estimate the respiratory frequency, $f_R(t)$, the respiratory signal was downsampled at 4 Hz, its TF spectrum was estimated by SPWVD and the algorithm described in Sec. 2.2 was applied in the entire TF domain, i.e. $\Omega_R^\alpha = \mathbb{R}^2$.

4. Results

The SPWVD was estimated with a kernel which gave a TF resolution of about (12s, 41mHz). These values correspond to the widening of spectral components which are ideally perfectly concentrated along a line in time or frequency direction. The TF domain was discretized in steps

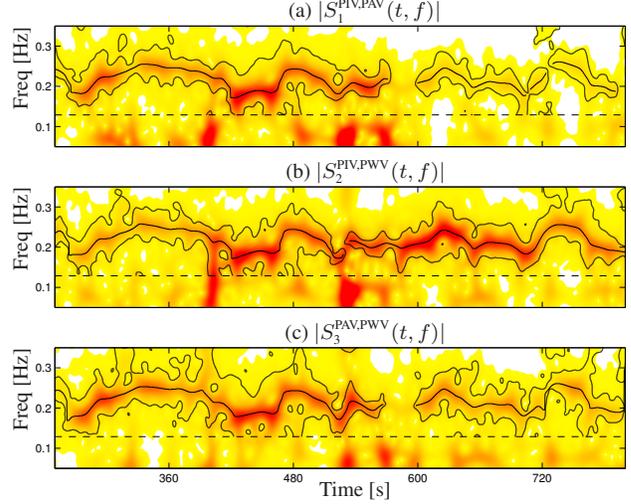


Figure 2. Cross TF spectra between: (a) PIV-PAV signals, (b) PIV-PWV signals, (c) PAV-PWV signals. Instantaneous frequencies $f_k^{ij}(t)$ are reported in black lines. Black contours encircle the TF regions of the respiration-related component $\Omega_{ij,k}^\alpha$. Horizontal lines represent f_M .

of 0.25 s and 1 mHz.

An illustrative example of the proposed algorithm is shown in Fig. 2–3. Figure 2 depicts the magnitude of the cross TF spectra, $|S_k^{ij}(t, f)|$, where the instantaneous frequency of the respiration-related component, $f_k^{ij}(t)$, are reported in black line. Regions $\Omega_{ij,k}^\alpha$, with $\alpha = 5\%$, are encircled by black contours and were bounded by $f_M = 0.13$ Hz. In the TF regions in which the local coupling was not statistically significant, $f_k^{ij}(t)$ was not estimated.

Figure 3 shows that, although in the considered intervals the respiratory rate was highly non-stationary, $\hat{f}_R(t)$ followed $f_R(t)$ with extremely low estimation error, whose median \pm interquartile range was 0.00 ± 4.88 mHz.

For a given subject s , the estimation error was estimated in mHz, as $E_s(t) = (\hat{f}_R(t; s) - f_R(t; s)) \cdot 1000$, and in relative unites, as $E_s(t) = (\hat{f}_R(t; s) - f_R(t; s)) / f_R(t; s)$. Global results are given in the table 1 as:

$$E_{\text{MED}} = \text{median}_s(\text{median}_t(E_s(t))) \pm \text{iqr}_s(\text{median}_t(E_s(t))) \quad (8)$$

$$E_{\text{IQR}} = \text{median}_s(\text{iqr}_t(E_s(t))) \pm \text{iqr}_s(\text{iqr}_t(E_s(t))) \quad (9)$$

where “median” and “iqr” stand for median and interquartile range, and are first estimated across time and then across subjects. In table 1, the amount of time during which the respiratory rate was not estimated (NE), i.e. $\hat{f}_R(t) = \emptyset$, is also reported. As expected, by decreasing α , the estimation error decreased and NE increased.

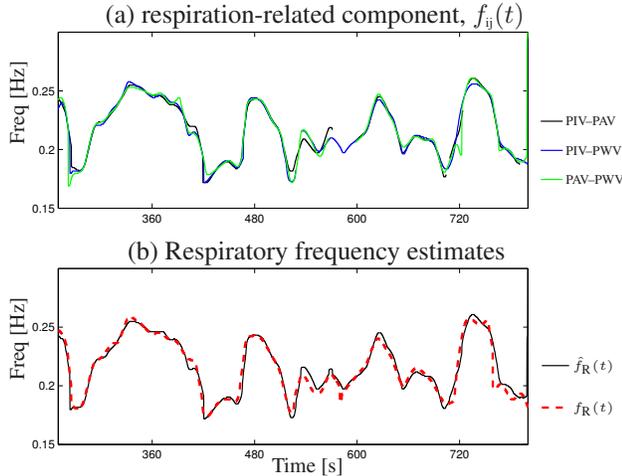


Figure 3. (a) Instantaneous frequencies $f_k^{ij}(t)$ estimated in Fig. 2. (b) Respiratory rate, $f_R(t)$, and estimated respiratory rate $\hat{f}_R(t)$.

Table 1. Estimation error, as in (8)–(9). NE: intervals where $\hat{f}_R(t)$ was not estimated.

		$\alpha = 10\%$	$\alpha = 5\%$	$\alpha = 1\%$
		med \pm iqr	med \pm iqr	med \pm iqr
E_{med}	[mHz]	0.00 \pm 1.95	0.00 \pm 1.95	0.00 \pm 1.22
E_{iqr}	[mHz]	8.79 \pm 7.20	7.81 \pm 6.10	6.84 \pm 4.52
E_{med}	[%]	0.00 \pm 0.76	0.00 \pm 0.75	0.00 \pm 0.39
E_{iqr}	[%]	3.63 \pm 4.29	3.00 \pm 4.03	2.69 \pm 3.56
NE	[%]	2.39 \pm 6.35	5.70 \pm 11.08	14.03 \pm 25.00

5. Discussion

In recent years, much effort has been put in the design of methods to indirectly estimate the respiratory rate from the PPG signal [4]. The presented methodology was shown to provide a continuous tracking of non-stationary respiratory rate with very high accuracy. The peculiarities of this method are (i) the use of different respiration-related features derived from the PPG signal, instead of the PPG signal; (ii) the use of TF analysis with high TF resolution; (iii) the use of coherence analysis to localize specific time-varying respiratory spectral bands and to perform a sort of validation of the estimates. It is worth noting that although in this paper we used as respiration-related features the PIV, PAV and PWV signals, the presented framework is a general one, and it offers the possibility of including more respiration-related features.

In this framework, the parameter α , Δt and Δf control the trade-off between the accuracy of the estimation and the amount of time during which the algorithm provides $\hat{f}_R(t) \neq \emptyset$. For instance, in those situations in which higher accuracy is more important than obtaining continuous es-

timates, $\alpha = 1\%$ can be used.

Another important characteristic of the algorithm is the possibility of estimating respiratory rate for $f_R(t) < 0.15$ Hz (7 subjects had $f_R(t) < 0.15$ Hz for at least one minute). The indirect estimation of respiratory rate for $f_R < 0.15$ Hz is particularly challenging since in this case the spectral range of the respiratory signal overlaps with that of other cardiovascular mechanisms (as Mayer wave). Although low respiratory breathing is a common physiological condition, many methods for the indirect estimation of the respiratory rate from the PPG were not tested at these frequencies [4]. In this methodology, accurate estimate of $f_R(t)$ were obtained also for low respiratory rate owing to coherence analysis and to the signal-dependent definition of Ω_{ij}^k .

In contrast to other studies [4], the respiratory rate was estimated in spontaneous breathing subjects during an autonomic test which induces quick changes in the cardiovascular variability. This is one of the most challenging conditions, since both the respiratory rate and the PPG signal are highly non-stationary.

In conclusion, during non-stationary conditions and in spontaneous breathing subjects, the described methodology gave robust and accurate respiratory rate estimates from the PPG signal, also for $f_R(t) < 0.15$ Hz.

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