# A Simple 2D Whole Heart Model for Simulating Electrocardiograms

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### **Abstract**

A simplified 2D whole heart model (2DWHM) which simulates the Electrocardiogram (ECG) accurately is presented. Although extremely detailed whole heart 3D models are available, they are computationally expensive. On the other hand most of the 2D cardiac models are homogeneous models aiming at modeling activation propagation in a small patch of cardiac tissue; they are not meant to be whole heart models. A two-dimensional heterogeneous "whole heart" model consisting of an array of specialized cardiac cells, with appropriate anatomical distribution, interacting via gap junction conductance (GJC) is envisioned to be a midway solution to this problem. The proposed 2D whole heart model includes various key components of the electrical conduction system of the heart including the SA (Sino atrial) node, fast conducting inter-atrial pathways, slow conducting AV (atrio-ventricular) node, Bundle of His, Purkinje network, and atrial and ventricular myocardial cells. Atrial and ventricular myocardial cells are modeled by Aliev-Panfilov (AP) two-variable model proposed for cardiac excitation. The auto rhythmic cells are modeled by Fitzhugh-Nagumo (FN) model operated in the oscillatory mode. In addition to normal ECG, the model also reproduces AV conduction blocks.

## 1. Introduction

Different modelling approaches are used in cardiac electrophysiology to understand the cardiac physiology and pathophysiology [1]. Each model is devised based on the purpose for which they are formulated and hence the complexity varies. Several microscopic and macroscopic cardiac electrophysiological models are available in literature[2, 3]. Extremely detailed whole heart 3D models are available, but they are computationally expensive. On the other hand most of the 2D cardiac models are homogeneous models aiming at modeling activation propagation in a small patch of cardiac tissue to explain propagation in the case of arrhythmia conditions like fibrillation, myocardial ischemia, infarction etc[4].

The proposed 2DWHM combines both microscopic and macroscopic aspects. At microscopic level the model uses simplified single cell models, whereas at the macroscopic level the model captures whole heart dynamics and simulates the ECG waveform.

# 2. The proposed 2D whole heart model

The key electrical activities of the cardiac conduction system are captured in the proposed 2D model which is based on a simplified two-dimensional representation of the human heart.

#### 2.1. Anatomical structure

The 2D geometry contains SA nodal cells, atrial myocytes, fast conducting intra-atrial pathways, AV nodal cells, Bundle of His cells, Purkinje cells and ventricular myocytes (fig.1). The 2D whole heart model of size 200 X 120 is labeled vertically into two regions separated by a low conductance band depicted by a dark line: the upper region representing the atria and the lower one representing the ventricles (fig 1). The sizes of the atrial and ventricular regions are 80X120 and 120X120 respectively. The low conductance band (dark line) separating atria and ventricles prevent the direct propagation of impulse from the atria to the ventricles other than via the AV node. To create asymmetry between the right ventricle and the left ventricle the Bundle fiber is positioned towards the right ventricle (left ventricle is thicker than the right ventricle). The Purkinje system is not modeled in its detailed arboreal structure but as two branches - one each to two ventricles (fig. 1).

In the proposed 2D whole heart model each cell is connected to its eight adjacent neighbors through gap junction connections. At each time step the state of a particular cell is affected by the states of cells connected to it. The strength of the connection can be varied by the gap junction conductance (GJC) values. If the connection strength is varied in different directions the direction of spread of the propagation can be varied. This can account for the fiber orientation and anisotropy present in the cardiac musculature in different regions of the heart.

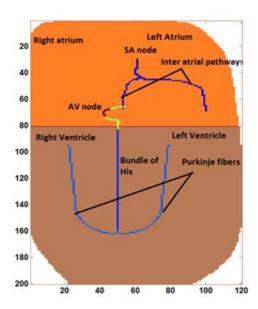


Figure. 1 Simplified schematic of the 2D structure of heart used in the simulation studies.

The action potential for a single cell is described using the equation

$$C_m \frac{dv}{dt} = -(I_{ion} + I_{stim}) \tag{1}$$

where v is the voltage across the cell membrane,  $C_m$  is the specific capacitance of the cell membrane,  $I_{ion}$  is the sum of all the individual ionic currents,  $I_{stim}$  is the externally applied stimulus current. In the 2D model since each cell is connected to eight neighbors,  $I_{stim}$  is the summation of all individual currents from the eight cells. The individual current to the cell at position (i,j) depends on the voltage difference and GJC between the cells.

## 2.2. Cell types

The proposed 2D cardiac network model is constructed using reduced cell models like the FitzHugh-Nagumo (FN) model [5] and the Aliev-Panfilov (AP) model [6].

FN model is a reduced form of classical Hodgkin Huxley model.

$$\frac{dv}{dt} = v(v-a)(1-v) - w + I_{stim}$$

$$\frac{dw}{dt} = \varepsilon(v - yw)$$
(2)

The fast variable v models the membrane potential of the cardiac cell and the slow variable w models the recovery of the membrane potential. The parameters  $\gamma$ , a and  $\varepsilon$  control the behavior of the model.

The action potential produced by the AP model has greater resemblance to the myocardial action potential with no super-repolarized state as FN model. It can also show the restitution property of cardiac tissue. The equations for the AP model are given as follows.

$$dv/dt = kv(v-a)(v-1) - vw + I$$

$$dw/dt = \gamma(v,w)(-w-kv(v-a-1))$$

$$\gamma(v,w) = \gamma_0 + \frac{\mu_1 w}{(v+\mu_2)}$$
The parameters  $k$ ,  $\gamma$ , and  $a$  relate to the standard FN

The parameters k,  $\gamma$ , and a relate to the standard FN model parameters and the parameters  $\mu_1$  and  $\mu_2$  calibrates to different restitution curves[6]. The additional parameter  $\gamma_0$  termed as refractoriness controls the action potential duration (APD) [7].

For reproducing positive T (ventricular repolarization) wave and reduced amplitude Ta (atrial repolarization) wave APD is varied in the myocardium by making use of the refractoriness parameter. It is split into three components and varied in the medial lateral direction for T wave. The variation is done moderately for the Ta wave

$$\gamma_0(i,j) = \gamma_{00} + \gamma_{01}(i) + \gamma_{02}(j) \tag{4}$$

 $\gamma_{00}$  is the maximum action potential duration of the myocyte,  $\gamma_{01}(i)$  is the apex-base variation and  $\gamma_{02}(j)$  is the transmural variation.

ECG signal is computed from the 2DWHM by the method described by Virag and Kappenberger [8]. In the 2D network of cardiac cells, each pair of adjacent cells forms an electric dipole of length 'd' and current density 'I' ( $\Delta V_m \times G$ ). The potential recorded at sites  $V_1$ ,  $V_2$  or  $V_3$  is equal to the sum of the contribution of all the dipoles in the network.

The contribution of each dipole V(i,j) in the vertical horizontal and diagonal directions is calculated as follows.

$$V(i,j) = \frac{(\Delta V_m \times G) \times d^2 \times \cos(\theta)}{4\pi R^2}$$
 (5)

where  $\Delta V_m$  is the potential between two adjacent cells, G the conductance between them, d the length of the dipole,  $\theta$  represents the angle between the position vector R and the dipole D.

$$V(k) = \sum_{i=1}^{200} \sum_{j=1}^{120} V_{ver}(i,j) + V_{hor}(i,j) + V_{diag}(i,j) + V_{diag}(i,j)$$
 (6)

V(k) can be  $V_1$ ,  $V_2$ , or  $V_3$  depends on the position vector R,  $V_{ver}(i,j)$  is the contribution of the dipole formed by the vertical pair of adjacent cells ,  $V_{hor}(i,j)$  is the contribution of the dipole formed by the horizontal pair of adjacent cells and the diagonal contributions are  $V_{diag1}(i,j)$ ,  $V_{diag2}(i,j)$ .

#### 3. Simulations

## 3.1. Normal ECG

The impulse is initiated in the SA node at the rate of about 80BPM and spreads throughout the atria resulting

in P wave. The delaying of the impulse in the AV node and Bundle of His is responsible for the PQ interval between the atrial depolarization and ventricular depolarization. The major contributor for the positive P wave is the asymmetric conduction of impulse in the right and left atria[9]. This is obtained by varying the diagonal GJC differently in the left atrium and right atrium (fig.2).

Once the impulse reaches the AV node it is delayed by the reduced GJC (0.08 $\mu$ S to 0.1 $\mu$ S) to produce the PR interval. The impulse slowly conducts through the bundle fibers and the purkinje fibers. The GJC (0.1nS to 1pS) between the bundle cells and the myocardial cells are kept low and impulse starts spreading into the myocardium once it reaches the left and right purkinje fibers. The GJ conductance between the Purkinje cells and ventricular myocardial cells is high (0.09  $\mu$ S -0.3 $\mu$ S) facilitating rapid spread of the impulse through the myocardium from apex to the base. The conductance among the Purkinje cells is made very high (0.3  $\mu$ S -0.7 $\mu$ S) compared to the ventricular cells (0.02  $\mu$ S - 0.07  $\mu$ S) to obtain a faster conduction and a sharp R wave.

Atria repolarize at the same time as ventricular depolarization and hence it is not shown in the ECG waveform. During the ventricular repolarization the APD of the ventricular myocardial cells is varied in the medial lateral direction as per equation (4) to obtain a positive T wave. This causes ventricles to repolarize simultaneously in all regions (fig. 3).

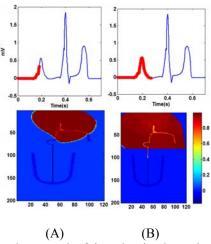


Figure 2. The spread of impulse in the atria produces atrial depolarization (P) wave. The red line on the ECG signal shows the instance of the spread of wave propagation in the 2D model (A) P wave traversed almost half of the atrial myocardium (180ms). (B) Atria fully depolarized (270ms).

## 3.2. AV block

Atrio-ventricular (AV) node is the only electrical

connection between the atria and ventricles through which the impulse from the atria can pass into the ventricles. The AV node can be divided into three regions, the AN region, which connects to the atrial musculature, the central N, nodal region and the NH region which connects to the His bundle [10]. The N and NH region cells possess automaticity which allows the AV node to be the backup pacemaker of the heart.

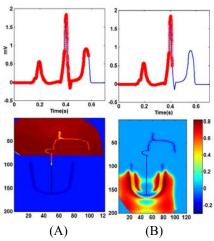


Figure 3. Ventricular activity (A) Ventricular depolarization wave 'QRS' (413ms). (B) Ventricular repolarization T wave (576ms).

PR interval in the normal ECG waveform is between 0.12s and 0.2s and if it is greater than 0.2 s with no failure in AV conduction, the condition is termed as 1<sup>st</sup> degree AV block. 2<sup>nd</sup> degree AV block can be classified into Mobitz types I and II. Mobitz type I is also called as Wenckebach block which is characterized by progressive PR prolongation and finally results in a missed beat. PR interval remains constant in Mobitz type II block [11]. In 2<sup>nd</sup> degree block atrial impulse is passed at the rate of X: (X-1) or X: 1 and if the condition is prolonged for a longer duration finally it can result in 3<sup>rd</sup> degree block where the atrial pulses are totally blocked from entering the ventricular conduction system.

In simulation studies (fig.4) 1st degree block is reproduced by decreasing the GJC of the AV bundle cells and bundle of his cells. In 2nd degree type 1 block (Wenckebach phenomenon) is characterized by progressive prolongation of PR interval before a dropped QRS complex. This can be obtained by increasing the heart rate, decreasing the GJC among the AV nodal cells and increasing the APD of AV nodal cells. By these variations the cells are stimulated in the relative refractory period (RRP) of the action potential which results in reduced amplitude and increased duration of the next AP which further results in similar AP's. This finally results in the Wenckebach phenomenon[10]. In 2nd degree type 2 Mobitz block the PR interval before and after the block remains the same. This condition is simulated by further

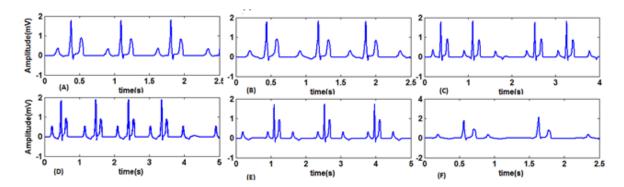


Figure 4. Different types of AV conduction blocks. (A) Normal ECG (B) First degree block where PR duration is increased more than 0.2sec(C) 2<sup>nd</sup> degree Type I (Wenckebach) 3:2, PR duration prolonged and the third P wave is blocked (D) 2<sup>nd</sup> degree Type I (Wenckebach) 5:4 PR duration prolonged in every beat and the fifth P wave is blocked (E) 2<sup>nd</sup> Degree (Mobitz) Type 2 PR duration is fixed before and after the block (F) 3<sup>rd</sup> degree (complete) block, all P waves are blocked from are entering the ventricles, ventricles contracting based on the escape beats from the ventricular conduction system.

decreasing the GJC so that the stimulus falls into the early refractory period of the AP thereby not stimulating the cell and the next stimulus falls after the RRP and stimulating the cell normally. In 3<sup>rd</sup> degree block the GJC is further decreased so that none of the impulse generated by the SA node is passed into the ventricles. The auto rhythmic cells in the ventricular conduction system produces escape beats.

### 6. Discussion

The proposed 2D WHM is based on a simple 2D geometry of the cardiac conduction system. The model is able to capture the dynamics of the electrical conduction system of human heart with heterogeneous cells. The ECG waveform generated by the model is based on discrete approach without using diffusion equations. This model presents a simple representation of the cardiac conduction system but robust enough to explain the key processes that contribute to the generation of the ECG waveform. This simulation study shows that the shape of the P wave and the ORS complex and T wave in the ECG signal depends mainly on the pattern of propagation of the activation waves which in turn depends on the cell type distribution GJC distribution, and APD heterogeneity in the cardiac musculature. It also reproduces AV conduction blocks by GJC and APD variation in the AV nodal cells.

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