Correlation Dimension as a Measure of the Atrial Fibrillation Capture during Atrial Septal Pacing

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Abstract

In the present model-based study, the correlation dimension (D2) was used as a measure of the effectiveness of atrial septal pacing during atrial fibrillation (AF). From ongoing simulated atrial fibrillation, instantaneous transmembrane potential maps were used as initial conditions for a rapid pacing from the septum area at pacing cycle length (PCL) expressed as a percentage of the AF cycle length. During the pacing, transmembrane potentials time series at four sites on the right/left atrial (RA/LA) walls were recorded and D2 was separately computed. The results indicated a lower D2 value (2.17±0.42) for PCL in the range 85-95% of AFCL compared to D2 (3.53±0.31) obtained for the other PCL values. Moreover, for PCL in the range 85-95% of AFCL the results indicated a better capture in the RA than in the LA (p<0.005). The overall results suggest that D2 can discriminate between different levels of atrial activity organization during AF pacing therapies. This makes possible the assessment of suitable PCL values leading to appropriate AF capture.

1. Introduction

Atrial fibrillation represents a top issue in world scientific research, mainly due to its complex nature and its incidence rate comparing to other cardiac arrhythmias. The main issues in AF data processing are the discrimination between AF groups using the electrocardiogram, the efficient monitoring of AF catheter ablation and the quantification of the influence of antitachyarrhythmia pacing in AF termination. In this context, computer modelling has gained importance in the development of therapeutic strategies for atrial fibrillation, mainly in the developing of novel pacing protocols. A computer model of AF offers the possibility to analyse various scenarios for the pacing algorithms, and also gives access to all atrial electrical activity (i.e. it is possible to record the simulated electrical activity on a grid covering the global atrial tissue and at the same time to observe the temporal and spatial evolution of depolarization waves and re-entrant waves on the whole atrial surface).

The potential of chaos-theory based approaches to discriminate between different types of fibrillation or to characterize cardiac time series both during normal or pathological conditions has been revealed in many papers [1-4]. Techniques derived from non-linear dynamics have been used for the quantification of atrial activity organisation in the context of AF therapies [5-7].

The main issue of the present paper was to investigate the potential of correlation dimension (D2) to discriminate between different levels of atrial electrical activity during AF pacing, by testing and validating it using a biophysical computer model of atrial fibrillation [8]. As the septum area was proved to be a good site for pacing [9], we aimed to test the effectiveness of D2 in searching for optimal pacing cycle length leading to a good AF capture.

In what follows, Section 2 briefly describes the biophysical modelling of atrial fibrillation and the pacing strategy, then the procedure for the assessment of AF capture by using correlation dimension estimation is presented. Section 3 shows and discusses the experimental results. Section 4 presents final remarks and concludes the paper.

2. Methods

2.1. Modelling pacing of atrial fibrillation

A programmed stimulation protocol was used to initiate atrial fibrillation in the biophysical model. The biophysical model was based on a homogenous tissue in which the Luo-Rudy model was adjusted to mimic electrical remodelling as observed in permanent AF, by setting the channel conductance GNa, GK and Gsi to 16, 0.423 and 0.55 mS/cm\textsuperscript{2} respectively [8-10]. The simulated AF was characterized by multiple wavelets continuously changing in size and duration. From 10 minutes of ongoing AF, 10 instantaneous transmembrane potential maps (corresponding to different states of
electrical activity in the atrial tissue) were selected as initial conditions for the subsequent simulation of atrial septal pacing. For each initial condition, a rapid pacing was applied by injecting a stimulus current inside the cells located in the septal area. The rapid pacing strategy was designed according to [11], in which the computer modelling was used to search for optimal pacing algorithm leading to a local capture of AF. For each of the 10 instantaneous transmembrane potential maps selected as initial conditions, a rapid pacing from the septum area with PCL in the range 61-99% of the AF cycle length (AFCL) with 2% increments was applied. The AF cycle length, computed out of 10 minutes of simulated AF, was AFCL=72ms.

2.3. Assessment of AF capture by using correlation dimension

The correlation dimension characterizes the self-similar structures and geometric properties of chaotic attractors. This dimension is related to the minimum number of variables needed to model the dynamics of the attractor. Lower values of the correlation dimension correspond to less complex dynamics. For correlation dimension estimation we used the procedure proposed by Grassberger and Procaccia [12].

The first step in D2 estimation is to reinterpret the particular time series as an m-dimensional object. One can reconstruct the attractor from a single scalar time series \( X = \{x_1, x_2, ..., x_N\} \) using the time delay vectors:

\[
y_k^{(m)} = (x_k, x_{k+1}, ..., x_{k+(m-1)\tau}), k = 1, 2, ..., N-(m-1)\tau
\]

For an infinite, noise free, data series one can find an embedding dimension \( m \) (and any delay \( \tau \)) for which the delay vectors yields a phase space that has the same properties as the one formed by the original variables of the system. As in practice we do not deal with infinite noise free data series, various methods have been suggested to estimate \( m \) and \( \tau \), but decisions are still often subjective and dependent upon the investigated signals. In the present study, the time delay \( \tau \) was taken as the time for the autocorrelation function to drop to \((1-1/e)\) of its initial value [13].

Thus, for each time series, an embedding delay time (selected as mentioned above) was used to reconstruct time-delay vectors by increasing the embedding dimension up to \( m = 20 \). For each successive embedding dimension, the correlation integral at distance \( r \) in the reconstructed phase space of embedding \( m \) was evaluated:

\[
C(m)(r) = \frac{1}{N_{ref}(N_{ref}-1)} \sum_{i=1}^{N_{ref}} \sum_{j=1, j\neq i}^{N_{ref}} \Theta\left(r - \|y_i^{(m)} - y_j^{(m)}\|\right)
\]

\( \Theta(...) \) is Heaviside function and \( N_{ref} \) is the number of the reference vectors used from the total \( N-(m-1)\tau \) embedding vectors.

From the double logarithmic plot of \( C(m)(r) \) as a function of the distance \( r \), we estimated the correlation

![Figure 1](image1.png)

**Figure 1.** Top panel: Atrial electrical activity dynamics after 50 seconds of pacing for PCL equal to 83% and 85% of AFCL, respectively. Bottom panel: 2-sec of transmembrane potential as measured in middle of the right atrial anterior wall.
dimension \( D_2^{(m)} \) for embedding dimension \( m \) as the slope of correlation integral in the scaling region. Thus, the correlation dimension was evaluated as a function of embedding dimension by calculating it at successively higher values of the embedding dimension. In the case when \( D_2^{(m)} \) saturated as a function of embedding dimension \( m \), this saturation value was taken as the correlation dimension.

In order to avoid situations when the correlation dimension would present no convincing saturation value, in what follows our results on \( D_2 \) were obtained by averaging the slopes of the correlation integral in the scaling region over embedding dimensions 15 to 20.

Finally, the optimal PCL leading to the best AF capture corresponded to the lowest \( D_2 \) value averaged over the sites of measurement. The lower \( D_2 \), the less complex is the behaviour of the analysed time series, and implicitly, the more organized is the atrial electrical activity.

3. Results

For each of the 10 initial conditions of the simulated AF, rapid pacing was applied during a period of 120 seconds. The transmembrane potentials time series at four sites located in the middle of the right/left atrial (RA/LA) posterior and anterior walls were recorded. Thus, for each PCL and each initial condition, we separately computed the correlation dimension for each recorded time series. The time series length was \( N=30000 \) samples (in order to remove possible temporal correlation between the state vectors, the sampling frequency was set at 250Hz). For the calculation of the correlation integral we used \( N_{ref} = 10000 \) reference embedding vectors.

The results indicated a \( D_2 \) value of 2.17±0.42 (mean ± SD) for PCL in the range 85-95% of AFCL. This represents a significant decrease (\( p<0.00001 \), K-S test) compared with the value of \( D_2 \) (3.53±0.31) obtained for the other PCL values (i.e. a more organized atrial activity for PCL in the range 85-95% of AFCL). The results are graphically presented in Figure 2.

The numerical interval 85-95% of AFCL is not a strict interval. Intermediate values between the two previously mentioned correlation dimensions were also found for 83% of AFCL and 99% of AFCL, but the visual inspection of transmembrane potential maps revealed a larger number of anchored waves than for PCL in the range 85-95% of AFCL.

Additionally, for PCL in the range 85-95% of AFCL the results indicated a better capture in the RA than in the LA (\( p<0.005 \)) which may be due to the influence of LA anatomical obstacles on AF perpetuation. The statistical analysis between RA and LA measurement sites is presented in Figure 3.

4. Discussions

The main aim of the present model-based study was to investigate if it is possible to quantify the influence of anti-tachyarrhythmia pacing in AF capture by using the correlation dimension \( D_2 \). As correlation dimension estimation is very sensitive to specific input parameters such as sampling frequency, time delay or time series length, we had to perform many experimental trials to select the most appropriate parameter values. During AF, \( D_2 \) did not present a clear saturation (as a function of the embedding dimension) which means that atrial fibrillation cannot be associated with a low-dimensional chaotic dynamical system. On the other hand, based on the obtained results, it can be concluded that the correlation dimension can discriminate between different levels of atrial activity organization during AF pacing therapies. Therefore, it could be used as a non-invasive discriminating metric in automatic assessment of AF capture during pacing. The conclusions are well
supported by the visual inspection of the transmembrane potential maps during simulations (the lower values of D2 correspond to a periodic pattern for the transmembrane potential maps with a large portion of atrial tissue controlled by pacing - except for reentrant wavelets still presented in the area distant from the septum) and also by previous studies [6], [11].

Moreover, due to the fact that the results were based on measurements obtained from a limited number of sites, D2 becomes more valuable for clinical applications in which the number of measurement sites is usually limited.

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