Spectral Analysis of Electroanatomical Maps for Spatial Bandwidth Estimation as Support to Ablation

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Abstract

Intracardiac electrograms (EGM) in Cardiac Navigation Systems (CNS) and in Electrocardiographic Imaging provide relevant information on the arrhythmia mechanism for supporting ablation. In our previous work, we proposed Manifold Harmonics Analysis (MHA) for establishing the spatial sampling rate in ElectroAnatomical Maps (EAM) accounting for anatomical and bioelectrical features (e.g., voltage or activation time). Here, we propose a theoretically founded method for spectrum representation in terms of spatial frequencies from MHA, which can determine the minimum number of EGM registered at different spatial positions for accurate EAM with a cut-off spatial bandwidth. The EAM spectrum magnitude is obtained by cross-correlation between the original spatial anatomical and bioelectrical features, and the corresponding coefficients projected onto the manifold harmonic basis. The cut-off spatial frequency is computed according to a threshold value \( TH \in [0,1] \), accounting for the EAM reconstruction quality. TH was scrutinized in high quality anatomical meshes from tomography images, and in simulated and real EAM from CNS. Experiments showed that \( TH > 0.98 \) is required to obtain accurate both anatomical meshes and EAM. Strong dependence was shown on EAM with the cut-off spatial frequency in terms of the arrhythmia mechanism.

1. Introduction

Electrical and anatomical maps (EAM) are used to visualize both the anatomical cardiac structure and electrophysiological (EP) features (activation sequence or voltage amplitude) projected onto the reconstructed anatomy during EP studies [1]. These maps are built by using Cardiac Navigation Systems (CNS), which acquire the three-dimensional spatial coordinates of the catheter within the heart and in real time. While the detailed anatomical shape of the map is obtained quickly with a multielectrode catheter or using computerized tomography (CT) or magnetic resonance images, the electrical information is only acquired in several locations where the electrogram (EGM) is recorded during 2.5 seconds, so that the larger the number of EGM acquired, the better accuracy of the EAM. These maps help to define both the arrhythmia mechanism and ablation targets, either in focal and reentrant arrhythmias, or in atrial fibrillation [2].

Given that the creation of the EAM is usually driven by heuristic considerations from electrophysiologists, it would be desirable to have a principled criteria about the number of spatial locations that are necessary for yielding accurate enough EAM during EP studies. Therefore, the goal of this work is two-fold: (1) to propose a theoretical method for spectrum representation in terms of spatial frequencies; and (2) to determine the minimum number of EGM per area units by establishing a cut-off spatial bandwidth (SB) for accurate EAM sampling. Hence, the ablation success probability and the procedure duration could be improved and reduced, respectively.

Fourier spectral analysis techniques have been used as a transformation from uniformly sampled Hilbert signal spaces to the frequency domain, where a clear cut-off criteria can be established with the Nyquist limit [3]. However, their application to EAM and meshes is not straightforward, due to their irregular sampling. A number of spectral mesh processing methods have been proposed in computer graphics, including clustering, mesh segmentation, remeshing, surfaces reconstruction, and texture mapping [4]. Whereas none of them can be straightforwardly applied to EAM analysis, our work is founded on the previous theoretical contributions in [5], where an approximation was proposed for the Fourier transform (FT) on meshes based on Manifold Harmonics Analysis (MHA).

The use of MHA for EAM analysis still has several limitations. A clear quantification of the spatial frequency and of the spectral content magnitude has not been defined to date, hence the usual considerations on spectral density estimation in a Fourier sense are not usable with that formu-
lation. In addition, MHA was defined for the geometry of meshes, but it did not account for the analysis of a numerical scalar field (EP feature) associated to the mesh vertices. Here, we propose a theoretical founded method for spectrum representation from MHA, yielding an estimation of the cut-off spatial frequency in terms of the reconstructed EAM quality. This method can be applied to EAM containing both anatomy and features information.

In the next section, we summarize the main principles of MHA and present the proposed method. In Section 3, we present the results samples per area units and the spectrum representation for the anatomy of meshes obtained from CT images, and for both features and anatomy in simulated and real EAM.

2. Spatial bandwidth on EAM

Although FT has been used for spectral analysis, its application to irregularly sampled meshes is not straightforward. FT decomposes a signal into a linear combination of the Laplacian Operator (LO) eigenvectors. The extension of LO to the discrete triangular mesh used in this work was proposed in [5], by defining the Laplace-De Rham operator (LRO) for 0-forms as

\[
\Delta = -\frac{1}{d} \star d
\]

with \(\star\) the Hodge star, \(d\) is the exterior derivative, and the coefficients of matrix \(\Delta\) are:

\[
\Delta_{ij} = -\cotan(\beta_{ij}) + \cotan(\alpha_{ij}) \sqrt{|v_i||v_j|} \]

\[
\Delta_{ii} = -\sum_{j} \Delta_{ij}
\]

with \(v_i = (x_i, y_i, z_i)\) and \(v_j = (x_j, y_j, z_j)\) being the vertices linked by an edge; \(\beta_{ij}\) and \(\alpha_{ij}\), the opposite angles to the edge between \(i\) and \(j\); and \(|v_i|\), the area of the Voronoi region of vertex \(v_i\) in its 1-ring neighborhood.

The couple \(\{H^k, \lambda_k\}\) are the eigenvectors and eigenvalues of \(\Delta\), which correspond to the orthonormal basis, called Manifold Harmonic Basis (MHB), and to frequencies \(\omega_k = \sqrt{\lambda_k}\), respectively. The frequencies are inversely related to the edge lengths, for instance, the maximum (Nyquist) frequency of the mesh corresponds to the half of the inverted minimum edge length. The projection of vertices onto the basis function is the Manifold Harmonic Transform (MHT), and the coefficients can be computed by

\[
\hat{a}_k = \sum_{i=1}^{n} v_i H^k_i,
\]

where \(n\) is the number of vertices. The mesh can be reconstructed back by using the inverse MHT, i.e., \(\hat{v}_i = \sum_{k=1}^{m} a_k H^k_i\), where \(m\) is the number of coefficients used for the reconstruction.

This previously described MHA is here extended to cardiac EAM, by including up to the fourth dimension, corresponding to the cardiac EP feature in EAM. The couple \(\{H_k, \lambda_k\}\) is obtained now by computing the eigenfunctions of \(\Delta\), whose vertices are now defined as \(u_i = (x_i, y_i, z_i, h_i)\), where \(h_i\) is the EAM cardiac feature (such as activation time, unipolar or bipolar voltage) and \((x_i, y_i, z_i)\) are coordinates in the vertex \(u_i\). A normalization was used for \(u\) to zero mean and standard deviation equaling the average of the standard deviation of all dimensions, to avoid distortion just due to cardiac feature and spatial coordinates having different physical magnitude.

Once MHT is computed and extended coefficients \(\hat{a}_{ik}\) are obtained, the steps for estimating the spectrum, the cut-off frequency, and the SB, are the following:

1. The \(j\)-th element of the spectrum is computed as the normalized correlation coefficient \(S_j\), between the \(j\)-th el-
Figure 2. Spectrum (zoom) of a real unipolar (a) and bipolar (b) EAM in a LVT. From left to right: original and reconstructed for $\text{TH}=0.98, 0.99, 0.995$ in the unipolar (c) and bipolar (d) EAM.

Table 1. Average ± standard deviation for the SB (samples per unit area) of the anatomical meshes in both left ventricles and atria.

<table>
<thead>
<tr>
<th>Chamber</th>
<th>TH=0.98</th>
<th>TH=0.99</th>
<th>TH=0.995</th>
</tr>
</thead>
<tbody>
<tr>
<td>Atria</td>
<td>0.08 ± 0.02</td>
<td>0.11 ± 0.03</td>
<td>0.19 ± 0.03</td>
</tr>
<tr>
<td>Ventricles</td>
<td>0.06 ± 0.01</td>
<td>0.08 ± 0.02</td>
<td>0.13 ± 0.02</td>
</tr>
</tbody>
</table>

Table 2. Average ± standard deviation for the SB (samples per unit area) of simulated activation EAM in a SR and a FL, in both RA and LA.

<table>
<thead>
<tr>
<th>EAM</th>
<th>TH=0.98</th>
<th>TH=0.99</th>
<th>TH=0.995</th>
</tr>
</thead>
<tbody>
<tr>
<td>SR-RA</td>
<td>1.37 ± 0.03</td>
<td>1.78 ± 0.28</td>
<td>2.44 ± 0.05</td>
</tr>
<tr>
<td>SR-LA</td>
<td>1.38 ± 0.02</td>
<td>2.06 ± 0.09</td>
<td>2.84 ± 0.08</td>
</tr>
<tr>
<td>FL-RA</td>
<td>1.57 ± 0.06</td>
<td>2.26 ± 0.29</td>
<td>3.10 ± 0.08</td>
</tr>
<tr>
<td>FL-LA</td>
<td>1.20 ± 0.11</td>
<td>2.05 ± 0.09</td>
<td>2.66 ± 0.14</td>
</tr>
</tbody>
</table>

Table 3. SB (samples per unit area) of a real bipolar and unipolar voltage EAM with 4783 vertices for a LVT.

<table>
<thead>
<tr>
<th>EAM</th>
<th>TH=0.98</th>
<th>TH=0.99</th>
<th>TH=0.995</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bipolar</td>
<td>0.16</td>
<td>0.27</td>
<td>0.38</td>
</tr>
<tr>
<td>Unipolar</td>
<td>0.12</td>
<td>0.23</td>
<td>0.36</td>
</tr>
</tbody>
</table>

between reconstructed and original meshes, which can be quantified by the accumulated normalized correlation coefficient for each $j$-th element of $\mathbf{u}$, as follows,

$$C_j(w_m) = \frac{\text{cov}(\hat{u}_m^j, u^j)}{\text{var}(\hat{u}_m^j)\text{var}(u^j)}$$

(3)

where $\hat{u}_m^j$ is the $j$-th element of $\hat{u}_m$.

3. Set a threshold value $\text{TH} \in [0,1]$ to select the cut-off frequency $w_c$ considering $C_x(w) > \text{TH}$, $C_y(w) > \text{TH}$, $C_z(w) > \text{TH}$, and $C_h(w) > \text{TH}$. Note that the lowest spectrum components provide the general shape of the mesh according to the representation of $S(w)$.

The cut-off frequency $w_c$ provides the SB, i.e., the number of samples per area unit, required to have the accuracy set by the threshold value TH.

3. Results

**SB for anatomical meshes.** Detailed meshes were used for validation of the proposed methodology in anatomy, which were obtained from the segmentation of left atrial and ventricular endocardium in CT images. Specifically, six left atrial and ventricular endocardia from images were segmented by using a region growing method and transformed into meshes with hundred of thousands of vertices [6]. Given the high computational burden of the
MHT, anatomical meshes were irregular and randomly subsampled with 5000 vertices. Then, atrial and ventricular meshes were transformed into a spectral domain by computing their MHT, and the SB was obtained according to the proposed methodology.

Figure 1 shows the spectrum and the cut-off frequencies for several TH, and the reconstructed meshes for an atrium and a ventricle. Higher cut-off frequencies were required for the atrium than the ventricles in the same TH due to the borders (high frequencies) generated by the shape of the veins. Table 1 shows the SB with 50 realizations of each atrium and ventricle, with TH= 0.98, 0.99, 0.995. The required SB for TH=0.995 was near the double than for TH=0.98, but the visual reconstruction was quite similar for both (see Figure 1 (c) and (d)). In addition, the difference between the original mesh and the reconstructed meshes for each TH was measured with the mean-squared error (MSE), and a significantly reduction was yielded from TH=0.98 . Therefore, TH=0.98 can be considered adequate enough for building the anatomy.

**SB for EAM.** SB was obtained for both anatomy and features in simulated activation EAM for two rhythms, a sinus rhythm (SR) and a flutter (FL) in the right atrium (RA) [7,8]. EAM meshes were again irregular and randomly subsampled with 5000 vertices, and they were transformed into a spatial frequency space to estimate the SB. Table 2 shows the SB with 50 realizations for several subsampling vertices of EAM, TH=0.98, 0.99, 0.995. As expected, the highest SB was obtained for FL in the RA due to border effects in the feature space, such as early-meeets-late or very defined scars, whose spectrum will tend to be infinite.

A real unipolar and bipolar voltage amplitude CNS EAM of a left ventricular tachycardia (LVT) was analysed by following the proposed methodology. Table 3 shows the SB associated to the bipolar and unipolar voltage EAM in a mesh with 4783 vertices (not subsampled). Given that bipolar EGM usually required higher bandwidth than unipolar, consistent results were obtained for SB, slightly higher SB (see Figure 2) was required for bipolar than unipolar EAM. The MSE was also computed for the original EAM and the reconstructed meshes for each TH and it was also yielded a significantly reduction of the MSE from TH=0.98, hence we conclude that TH=0.98 can be appropriate to reconstruct the unipolar and bipolar EAM, and a post-processing step for boundary enhancement could be used to improve the map quality.

### 4. Conclusions

We have proposed the representation of the spectrum of EAM in CNS, in a Fourier analysis sense, by using appropriate modifications of MHA. This allows us to move towards well-known concepts of spectral analysis and their use in EAM, such as the Nyquist frequency, yielding a cut-off frequency which provides the SB required for a given map quality and accuracy. Previous mesh analysis using MHA has been extended for its usefulness in EAM using not only the anatomy, but also the cardiac feature associated to each vertex. Consistent results are obtained when analyzing atrial and ventricular meshes, both in complex simulations and in CNS real examples. The extended MHA allows us to quantitatively represent the EAM spectrum, yielding a principled estimation of their spatial frequency, and opening the field to a new set of tools for supporting the generation of EAM during EP studies.

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### References


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