A Geometric Deformable Model for Echocardiographic Image Segmentation

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Abstract

Gradient vector flow (GVF), an elegant external force for parametric deformable modals, can capture object boundaries from both sides. A new geometric deformable model is proposed that combines GVF and the geodesic active contour model. The level set method is used as the numerical method of this model. The model is applied for echocardiographic image segmentation.

1. Introduction

Deformable models have been extensively studied and used in medical image segmentation. There are mainly two kinds of deformable models: parametric deformable models and geometric deformable models. "Snakes" [1] are the most famous parametric deformable models, and the geodesic active contour model [2] is a typical geometrical deformable model.

In both kinds of deformable models, curves are deformed under the influence of the internal force, coming from within the curve itself, and the image based external force until a balance between them is reached. However, the representation of the curves is different. In parametric deformable models, the curves are expressed explicitly in their parametric forms. While in geometric deformable models, curves are implicitly expressed as level sets of a three-dimensional scalar function.

Gradient vector flow (GVF) [3] has been recently introduced as a new external force to enhance the parametric deformable model. In this study, we proposed a new geometric deformable model that combines the geodesic active contour model and GVF, and applied this model for echocardiographic image segmentation.

2. Background

A typical parametric deformable model can be expressed as

$$\frac{\partial \mathbf{x}}{\partial t} = \alpha \frac{\partial^2 \mathbf{x}}{\partial s^2} - \beta \frac{\partial^3 \mathbf{x}}{\partial s^3} - \nabla E_{ext}, \quad (1)$$

where $\mathbf{x}(s,t)$ is a curve treated as a function of time, α and β are model parameters, E_{ext} is the external energy function, and ∇ is the gradient operator. In an image $I(x,y)$ a common external energy function is chosen as

$$E_{ext} = -\left|\nabla[G\sigma(x,y) * I(x,y)]\right|^2, \quad (2)$$

where $G\sigma(x,y)$ is a 2D Gaussian function with standard deviation σ.

The traditional snake is limited by the typical external energy function shown above, and GVF [3] is introduced to enhance the model by replacing $-\nabla Eext$ in equation (1). Let $\mathbf{v}(x,y) = [u(x,y), v(x,y)]$ be GVF, and it can be obtained by minimizing the following energy functional

$$E = \iint \mu[(\frac{\partial u}{\partial x})^2 + (\frac{\partial u}{\partial y})^2 + (\frac{\partial v}{\partial x})^2 + (\frac{\partial v}{\partial y})^2] + |\nabla f|^2 |\mathbf{v} - \nabla f|^2 \quad (3)$$

where μ is a parameter which should be set higher with more noise, and f is the edge map, usually defined as

$$f = \left|\nabla[G\sigma(x,y) * I(x,y)]\right|. \quad (4)$$

Using calculus of variation, GVF can be obtained by solving the following Euler equations:

$$\mu \nabla^2 u - (u - \frac{\partial f}{\partial x})[(\frac{\partial f}{\partial x})^2 + (\frac{\partial f}{\partial y})^2] = 0 \quad (5a)$$

$$\mu \nabla^2 v - (v - \frac{\partial f}{\partial y})[(\frac{\partial f}{\partial x})^2 + (\frac{\partial f}{\partial y})^2] = 0 \quad (5b)$$

GVF is a very elegant external force, which can capture boundaries from the both sides. More details on GVF can be found in [3].

The geodesic active contour is defined as

$$\frac{\partial U}{\partial t} = g(c+\kappa)\left|\nabla U\right| + \langle \nabla U, \nabla g \rangle, \quad (6)$$

where U is a 3D scalar function, whose zero level set is the evolving curve, g mainly functions as the stop force when the evolving force arrives to the object boundary and defined as

$$g = \frac{1}{1+f^p} \qquad p = 1 \; or \; 2 \;, \tag{7}$$

c is a constant to increase the speed of convergence, and κ is the curvature given by

$$\kappa = \frac{\frac{\partial^2 U}{\partial y^2}(\frac{\partial U}{\partial x})^2 - 2\frac{\partial U}{\partial x}\frac{\partial U}{\partial y}\frac{\partial U}{\partial x \partial y} + \frac{\partial^2 U}{\partial x^2}(\frac{\partial U}{\partial y})^2}{((\frac{\partial U}{\partial x})^2 + (\frac{\partial U}{\partial y})^2)^{3/2}} \;. \tag{8}$$

In the model, $\nabla U . \nabla g$ attracts the curve to the object boundary.[2]

3. New geometric deformable model

The proposed geometric deformable model can be thought of as a modified geodesic active contour model for the vector field, GVF. Similarly the g function should stop the evolving curve around the boundary of the vector field. Based on equation (7), a new f function should be defined to obtain the gradient of the vector field, GVF. This procedure is similar to edge detection of color image. A technique similar as [4] is applied here.

Let **J** be a Jacobin matrix given by

$$\mathbf{J} = \begin{bmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{bmatrix} , \tag{9}$$

and define **D** as

$$\mathbf{D} = \mathbf{J}^T \mathbf{J} = \begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{bmatrix} . \tag{10}$$

It can be shown that the magnitude of the gradient of GVF can be defined as the largest eigenvalue of **D**. A small change is made here. We define the new function, f, as the difference subtracting the smallest eigenvalue from the largest eigenvalue. Let f_{GVF} be the new function, and it is easy to get that

$$f_{GVF} = [(d_{11}+d_{22})^2 - 4(d_{11}d_{22} - d_{12}d_{21})]^{1/2} . \tag{11}$$

And the new g function g_{GVF} is given as

$$g_{GVF} = e^{-f_{GVF}} . \tag{12}$$

Furthermore, since GVF can capture object boundary from both sides, we replace ∇g with -**v** to prevent the evolving curve from leaking through the boundary gaps. Now we can define the new geometric model as

$$\frac{\partial U}{\partial t} = g_{GVF}(c+\kappa)|\nabla U| + \langle \nabla U, \; -\mathbf{v} \rangle \tag{13}$$

4. Numerical methods

GVF can be calculated by applying the finite difference technique to solve the decoupled partial differential equations of 5a and 5b.[3] Central difference is used to calculate g_{GVF} and curvature κ.

Based on the level set method [5], the numerical solution of the proposed model (13) is given by

$$\begin{aligned} U_{ij}^{n+1} = U_{ij}^n + \Delta t\{ & g_{GVF}(i,j)\kappa_{ij}^n[(D_{ij}^{0x})^2 + (D_{ij}^{0y})^2]^{1/2} \\ & + g_{GVF}(i,j)\max(c,0)\nabla^+ + g_{GVF}(i,j)\min(c,0)\nabla^- \\ & - [\max(u(i,j),0)D_{ij}^{-x} + \min(u(i,j),0)D_{ij}^{+x}] \\ & - [\max(v(i,j),0)D_{ij}^{-y} + \min(v(i,j),0)D_{ij}^{+y}]\}, \end{aligned} \tag{14}$$

where

$$\begin{aligned} & D_{ij}^{-x} = \frac{U_{ij}^n - U_{i-1,j}^n}{\Delta x}, \; D_{ij}^{+x} = \frac{U_{i+1,j}^n - U_{i,j}^n}{\Delta x}, \\ & D_{ij}^{-y} = \frac{U_{ij}^n - U_{i,j-1}^n}{\Delta y}, \; D_{ij}^{+y} = \frac{U_{i,j+1}^n - U_{i,j}^n}{\Delta y}, \\ & D_{ij}^{0x} = \frac{U_{i+1,j}^n - U_{i-1,j}^n}{2\Delta x}, \; D_{ij}^{0y} = \frac{U_{i,j+1}^n - U_{i,j-1}^n}{2\Delta y}, \\ & \nabla^+ = [\max(D_{ij}^{-x},0)^2 + \min(D_{ij}^{+x},0)^2 + \max(D_{ij}^{-y},0)^2 \\ & \quad + \min(D_{ij}^{+y},0)^2]^{1/2}, \\ & \nabla^- = [\max(D_{ij}^{+x},0)^2 + \min(D_{ij}^{-x},0)^2 + \max(D_{ij}^{+y},0)^2 \\ & \quad + \min(D_{ij}^{-y},0)^2]^{1/2}. \end{aligned} \tag{15}$$

5. Segmentation of echocardiographic image

We applied the new geometric active contour model algorithm to an echocardiographic image sequence of the left ventricle with a short-axis view. We set the parameters as

$$\Delta x = \Delta y = 1, \Delta t = 0.5, c = -2.2, \mu = 0.4, \text{and } \sigma = 2.5.$$

Figures 1-5 show the convergence of the algorithm. After 45 iterations, the curve stops around the main boundary inside left ventricle cavity.

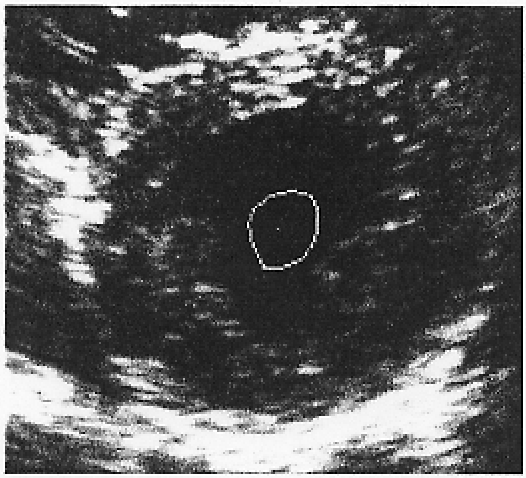

Fig 1. Initialization

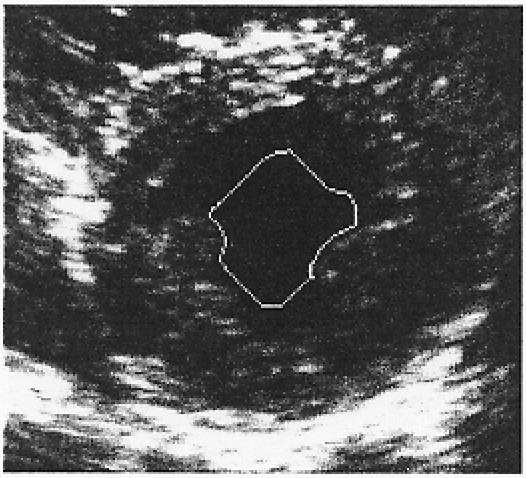

Fig.2 After 15 iterations

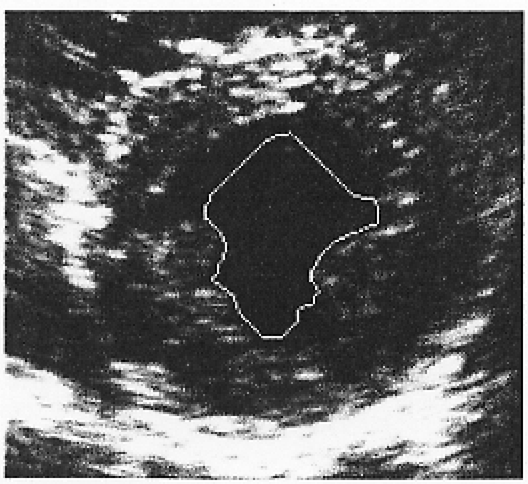

Fig.3 After 24 iterations

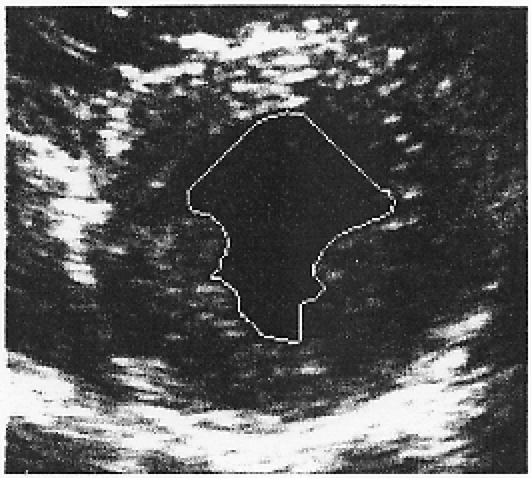

Fig.4 After 33 iterations

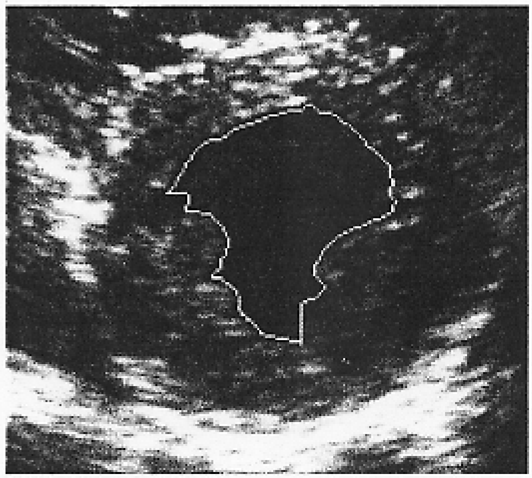

Fig.5 After 45 iterations

6. Conclusion

A new geometric deformable model is proposed via combining geodesic active contour model and gradient vector flow. Numerical method is realized by level set methods. The model is applied to the segmentation of echocardiographic image and shows quick convergence and accurate segmentation. The future research may lies in the extension of the model to three-dimensional for echocardiographic volumetric image analysis, such as left ventricle volume quantification and mitral valve structure extraction.

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