Pattern Complexity and Nonlinear Dynamics in RR-Sequences

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Abstract

The analysis of time series measured from nonlinear signals, may be performed either in the phase space or in the tie-domain. The Largest Lyapunov Exponent (LLE) characterizes exponential divergence of trajectories in the phase space; fractal analysis is able to describe the complex pattern of a given time series. To evaluate the relation between the dynamic behavior and pattern complexity of the inherent biological system, RR-interval sequences were derived from 24-hour Holter recordings performed in 55 healthy subjects (37 +/- 4 years, 34 males). Pattern fractal analysis (PFD) was computed on the basis of the measured length and diameter of the signal pattern. and LLE was evaluated by the Wolf algorithm. For each subject, the linear regression between computed PFD and LLE measures over the 24-hour period has been computed, extracting the correlation coefficient and the slope of the PFD vs. LLE relation. The strongest linear correlation between LLE and PFD indicates a tight link between the system dynamics and the pattern of the extracted signals. This link suggests the possibility of a direct evaluation of nonlinear dynamics, even over short time intervals, exploiting the computationally less expensive PFD.

1. Introduction

The analysis of heart rate variability (HRV) is currently performed either in the time domain, statistically measuring the overall variance of oscillations, or in the frequency domain, where spectral estimates define their frequency and amplitude.

Heart activity, vasomotion and respiratory function are characterized by an intrinsic complexity, likely related to the regulatory neural outflow dynamics, disrupted by disease, which has a deterministic chaotic nature as recently described [1], with clinical and even prognostic relevance [2]. Among all the "nonlinear" descriptors, most have limitations for the investigation of

biological time series, due to the long epochs required for the analysis and the heavy computational time needed.

Different extracted parameters are related to different aspects of signal nonlinearity. The LLE, computed after the reconstruction of the relative phase space, gives an index of dynamical complexity in the signal [3,4], whereas fractal analysis quantifies the signal propensity to space occupation, evaluating the morphological complexity exhibited by the pattern [5]. Nonlinear dynamics of the inherent biological signals is highlighted by the positive LLE, while the complexity of the RR pattern is singled out by PFD higher values.

To assess the efficacy of an original approach to fractal dimension computation reflecting the complexity of the signal described even over short periods, it has been compared with a recognized marker of nonlinearity, as the LLE over circadian RR interval series as derived from ambulatory ECG in a subset of healthy subjects.

2. Lyapunov exponents computation

Given a d-dimensional phase space there are d Lyapunov exponents which are related to the evolution of the axes of an infinitesimal d-sphere. If p_i is the ith axis the ith exponent is defined by:

$$\lambda_i = \lim_t \frac{1}{t} \log_2 \frac{p_i(t)}{p_i(0)} \qquad (1)$$

The Lyapunov exponents quantify the divergence of nearby orbits in the different directions of the phase space. In our study we are interested only in the largest Lyapunov exponent, defining the maximum orbit divergence.

To compute LLE from a time series, the phase space has been reconstructed from the RR interval series by the Takens method of delays [6]. We reconstructed the phase space vectors without interpolating the data choosing the sampling time as the mean RR period and using Takens formula, as if the RR series was sampled at regular intervals. The ith phase space vector is:

$$y(i) = [s(i) \quad s(i+\tau) \quad s(i+2\tau) \quad \Lambda \quad s(i+\tau(d-1))]$$

$$(2)$$

Where s is the RR series, d the phase space dimension and τ the delay (multiple of the mean RR interval). LLE was computed using the Wolf algorithm [3], a set of initial conditions (two nearby vectors in the phase space) is observed over time, LLE is evaluated by computing the divergence of the orbits (3) (see figure 1).
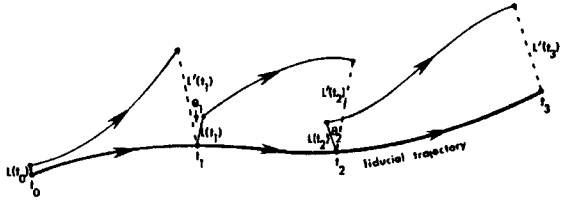

Fig. 1. The algorithm follows orbits in phase space.

$$LLE = \frac{1}{t_m - t_0} \sum_{k=1}^{M} \log_2 \frac{L'(t_k)}{L(t_{k-1})}$$

$$(3)$$

To verify the functionality of the algorithm it was applied to some chaotic series (logistic map, Henon map).

3. Fractal dimension analysis

The fractal analysis of the time series was performed with an original algorithm determining Pattern Fractal Dimension (PFD) [7], PFD (5) was derived from the classical Fractal Dimension formula (4) as defined by Katz [8] where L is the length of the pattern (i.e. the sum of the distances between successive points of the broken line), d is the diameter (i.e. the maximum distance between the first point and any other point of the pattern) see figure 2.

$$FD = \frac{\log(L)}{\log(d)}$$

$$(4)$$
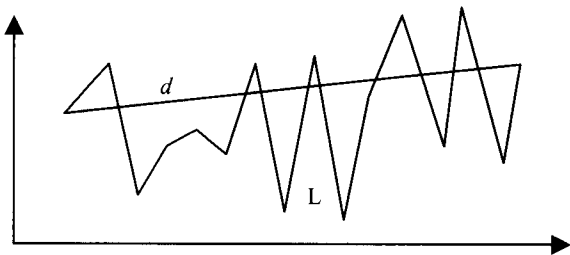

Fig. 2. The diameter d and the length L of the pattern.

The proposed modified algorithm is

$$PFD = 1 + \frac{\log(L/d)}{\log(n)}$$

$$(5)$$

where n represents the number of samples; the measure results to be independent from linear shifts, with values always greater than 1 and smaller than 2.

4. Subjects and methods

RR-interval series have been derived from 24-hour Holter recordings performed on 55 healthy subjects (34 males, 37+/-4 years, mean+-STD). Each extracted time series (long about 10e4 points) was analyzed over consecutive 1024-beat long windows for LLE and both 1024 and 256-beat for PFD, respectively. Both algorithms where developed entirely under MATLAB environment. The LLE algorithm requires the determination of some parameters: the phase space dimension d, the delay τ. To choose the time delay τ, in order to examine the nonlinear signal structure, we searched for the first minimum in the graph of average mutual information. To evaluate the embedding dimension d, according to the method proposed by Grassberger and Procaccia [9], we determined the correlation dimension of the attractors

The two algorithms were applied to the recorded 24-hour RR series, thus obtaining two series of findings for each subject. We applied Spearman's non parametric test on the series obtained with equal window length using the LLE values and the PFD values on 1024-beat windows. We averaged the 24-hour LLE and PFD values for each subject. Spearman's test has been applied on the whole population.

5. Results

After a few adjustments of the parameters the LLE algorithm gave correct results for the logistic map (6) for values of $R > 3.45$ see figure 3. The algorithm gave correct values also for the Henon map.

$$X_{n+1} = RX_n(1 - X_n)$$

$$(6)$$

For the RR time series considered, almost always, LLE values resulted positive (0.1011 +/- 0.031), suggesting the nonlinearity of the inherent dynamical system. For the same time series, the PFD values were greater than one and smaller than two (1.4328 +/- 0.066).
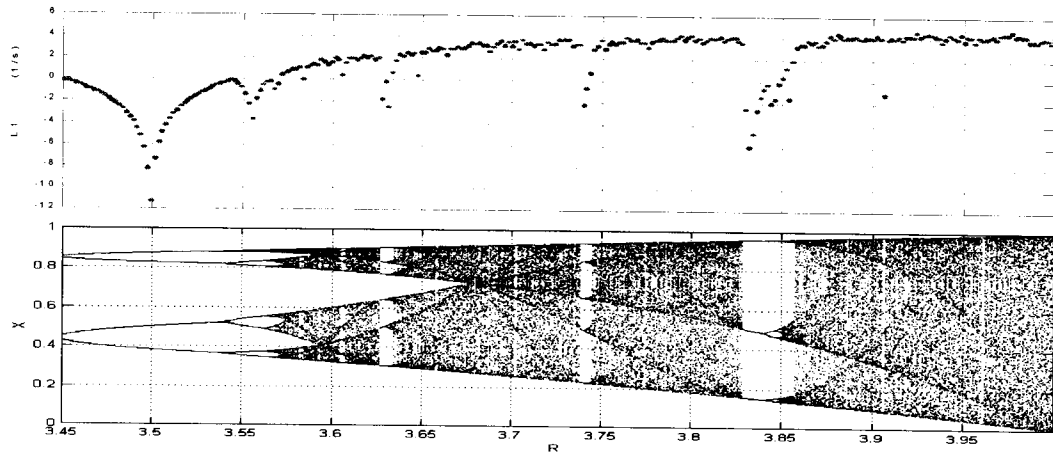
Fig. 3. Logistic map and LLE.

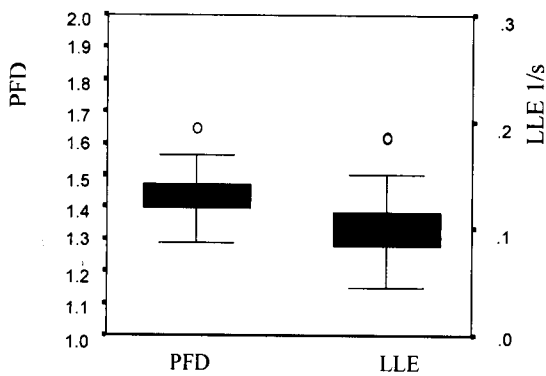The boxplots for LLE and PDF are reported in Figure 4.

Figure 4. Boxplot of PFD and LLE values, derived by RR sequences in the 55 healthy subjects.

Observing the values of LLE and PFD in the 24-hour period we noted a similar trend, as reported in Figure 5.

PFD and LLE computed on the same window of the same registration were plotted together on a scatterplot; nonparametric Spearman's test was applied to assess the correlation between the two measures, evidencing a high degree of correlation, as shown in Figure 6.

The correlation between the two measures was high also on the 24-hour mean values of LLE and PFD over the whole population (Spearman's ρ = 0.926, p<0.001, slope of LLE vs PDF relation = 0.434 with 95% CI: 0.385 –0.482).
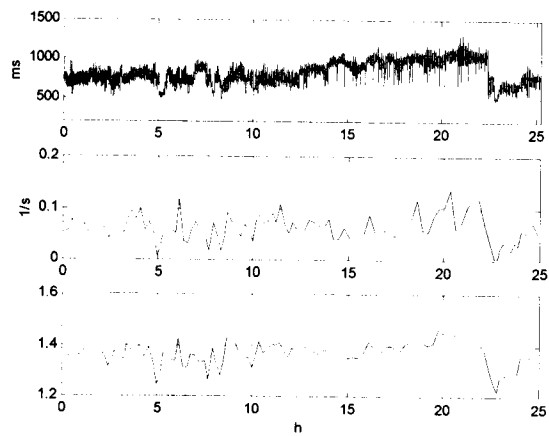

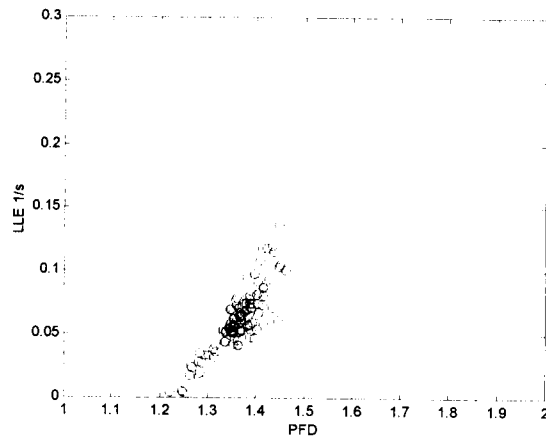Fig. 5. RR series, LLE and PFD in healthy subjects, over the 24-hour period.

Fig. 6. LLE vs PFD computed for the RR time series of an healthy subject. Spearman's ρ = 0.931, p<0.001; slope: 0.444, with 95% CI: 0.409-0.478.

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The correlation between the two measures was high also on the 24-hour mean values of LLE and PFD over the whole population, see scatter plot in figure 7.
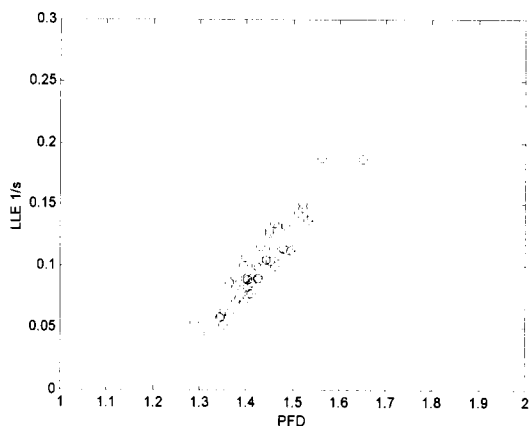

Figure 7. Correlation between 24-hour mean values of LLE and PFD. Spearman's $\rho = 0.934$, $p<0.001$; slope: 0.454, with 95% CI: 0.420-0.483.

6. Conclusions

Two different aspects of signals nonlinearity, i.e. the inherent orbit dynamics in the phase space and the convolutedness of the pattern, have been contemporaneously focused on the same acquired RR time series. While the positive values for LLE indicate a nonlinear dynamics, the greater than one values for PFD evidenced the pattern propension of space filling.

For each of considered healthy subject, a strong linear correlation between LLE and PFD has been found as the computing window was shifted along the 24-hour recordings.

The same result has been found when the mean value of all the time-varying values of PFD and LLE have been extracted for the 24 hours recordings.

The strong linear correlation between LLE and PFD quantifies an intimate link between the system dynamics and the pattern of the extracted signals ($r > 0.9$, $p <0.0001$). This link suggests the possibility of a direct evaluation of nonlinear dynamics, even v short time intervals, exploiting the computationally less expensive PFD.

The physiological dynamics of heart rate series over the day-night period in healthy subjects is characterized by complexity and dynamic chaoticity, well described by LLE and PFD give reproducible findings, with the convolutedness of pattern strictly related to the orbits divergence in the phase space.

References

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