

Hybrid Deformation Model of Myocardium

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Abstract

Computer models of the heart lead to a better understanding of the physiological and physical processes underlying each heart beat. Various models exist for simulating electrophysiology, excitation propagation, force development and deformation. Simulations can be used to support medical doctors in diagnostics, surgery planning and serve educational purposes. This work focuses on simulating mechanical aspects of a heart. Simulations with electrophysiological, excitation propagation and force development models were carried out. Force development was utilized as input to the mechanical model. A hybrid myocardial deformation model is introduced merging a spring mass system and a continuum mechanical model. Simulations with simple geometries and fiber orientation were conducted to display the models capabilities. Detailed analysis of the deformations of a patch yielded the expected behavior.

1. Introduction

Computers have become an essential tool in modeling, visualization and estimation of experimental results, thus saving time, resources and not conflicting with ethical objections. In cardiac research computer models and simulations lead to a better understanding of the physiological and physical processes. Simulations of the heart can be used for research, surgery planning and educational purposes.

In this work a simple ventricle model, realized as a half ellipsoid, was constructed taking anatomical and physical properties into account e.g. fiber orientation and electrical conductivity. Electrophysiological models, excitation propagation and force development models were applied resulting in a time course of force development for each voxel, which served as input for mechanical calculations. A hybrid deformation model was implemented based on a spring mass system enhanced by continuum mechanics based methods (Fig. 1).

This paper focuses on deformation simulations with the hybrid deformation model, carried out on a patch taken

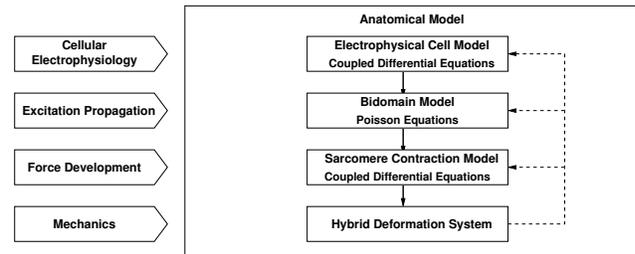


Figure 1. Overview of myocardial deformation simulation. The solid arrows show step by step progression of simulation. The dashed arrow represents the possibility of mechanical feedback.

from the simple ventricle model.

2. Materials and methods

In this work a simple ventricle model was constructed. The geometry consisted of a half ellipsoid representing a ventricle, where physical properties were implemented (Fig. 2). Electrophysiological simulations were carried out utilizing a model of Noble et al. [1]. The excitation propagation was modeled using the bidomain model [2]. Electromechanical coupling was implemented via exchange of intracellular concentration of calcium [3]. The electrophysiological simulations were done prior to mechanical deformation simulations and resulted in a time course of force development (Fig. 3).

A patch was extracted from the ventricle model to conduct mechanical deformation simulations. The patch consisted of $6 \times 5 \times 4$ voxels and was taken from the upper wall of the ellipsoid (Fig. 2). The force development calculated inside the geometric boundaries of the patch was used as input for the deformation simulation.

2.1. Hybrid deformation model

The deformation model was based on the spring mass system of Bourguignon et al. [4] enhanced by continuum mechanics based methods [5]. A spring mass system

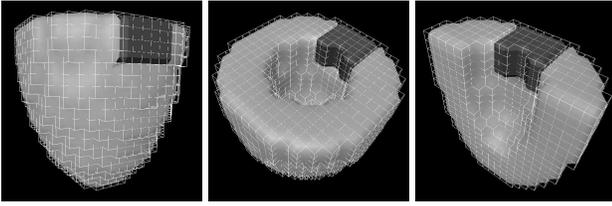


Figure 2. A half ellipsoid was rendered in a $20 \times 20 \times 20$ voxel lattice with edge length of 0.2 mm. A patch of $6 \times 5 \times 4$ voxels was extracted shown in dark gray, to conduct deformation simulations.

represents objects by masses and springs reproducing their physical quantities with appropriate points and constraints. This leads to a mesh structure approximating the object geometry. In this work a myocardial patch of $6 \times 5 \times 4$ voxels with fiber orientation varying from epi- to endocardium from -75° to 75° was represented as a volumetric mesh (Fig. 4). A cubic element was chosen as underlying mesh structure representing a cluster of cells. Each cell was formed by masses at the corners and springs aligned with the cubes edges (Fig. 5).

The myocardial properties were implemented as proposed by Bourguignon et al. [4], who utilized three linear anisotropic directional and eight linear volumetric springs. The first of three anisotropic springs were set in fiber direction at the center of each voxel. The remaining denoted sheet and sheet normal direction and were kept perpendicular throughout the simulation. Forces generated by springs were translated by linear interpolation functions to corresponding corner masses [6]. The isovolumetric springs were attached to the center and the corner masses of the voxel. This model was tested but simulations depicted that finding appropriate spring parameters was a time consuming task and modeling of nonlinear behavior of myocytes was impossible. Therefore, a continuum mechanics approach was chosen to form a hybrid model.

An exponential strain energy density function \mathbf{W} to

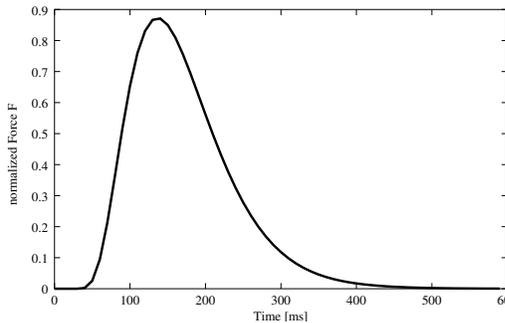


Figure 3. Time course of normalized force calculated with force development models for a voxel at the upper rim of the half ellipsoid.

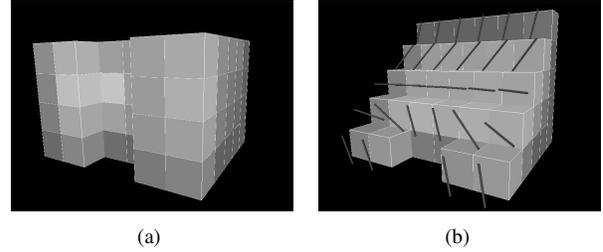


Figure 4. Myocardial patch built of $6 \times 5 \times 4$ voxels denoted by wireframe outlines (a). The patch is cut in a stairway to display fiber orientation (b). Fiber orientation varies from epi- to endocardium from -75° to 75° shown by black cylinders.

model anisotropic myocardial mechanics as described by Guccione et al. [7] was applied:

$$\begin{aligned} \mathbf{W} &= \frac{C}{2} (e^Q - 1) \\ Q &= 2b_1 (\mathbf{E}_{RR} + \mathbf{E}_{FF} + \mathbf{E}_{CC}) \\ &+ b_2 \mathbf{E}_{FF}^2 + b_3 (\mathbf{E}_{CC}^2 + \mathbf{E}_{RR}^2 + \mathbf{E}_{CR}^2 + \mathbf{E}_{RC}^2) \\ &+ b_4 (\mathbf{E}_{RF}^2 + \mathbf{E}_{FR}^2 + \mathbf{E}_{FC}^2 + \mathbf{E}_{CF}^2) \end{aligned}$$

Where C and b_i are constants chosen as described by Sachse [8]. Q is the representation of three-dimensional transverse isotropy with respect to fiber orientation dependent on the Green-Lagrange strain tensor \mathbf{E} . Indices F, C and R indicate fiber axis, cross-fiber in plane axis and radial axis.

This energy density function \mathbf{W} was used to calculate material stress starting from material strains. For each voxel the deformation gradient \mathbf{F} and Green-Lagrange strain tensor \mathbf{E} was determined. The second Piola-Kirchhoff stress tensor \mathbf{S} was derived and the Cauchy stress tensor $\boldsymbol{\tau}$ calculated:

$$\mathbf{S} = \frac{\partial \mathbf{W}}{\partial \mathbf{E}}, \quad \boldsymbol{\tau} = \mathbf{F} \frac{1}{\det \mathbf{F}} \mathbf{S} \mathbf{F}^T$$

The tensor $\boldsymbol{\tau}$ applied to the voxel surfaces led to a force acting on the corner masses. The masses were displaced

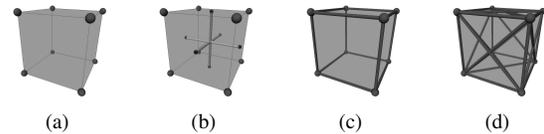


Figure 5. A cubic voxel of myocardial tissue is modeled with masses and springs. (a) Masses are denoted as spheres at the corner of a voxel. (b) Anisotropy is modeled with three springs, displayed as cylinders, which are located at the voxel center. They describe fiber, sheet and sheet normal orientation. (c) Structural and (d) surface springs were used for continuity of voxel.

and a new strain setup was achieved. This iterative process was continued until displacements were small.

The isovolumetric springs were replaced by isovolumetric constraints adopted from a Mooney-Rivlin model, whereby only the isovolumetric component of the Mooney-Rivlin strain energy density function \mathbf{W}_M was utilized:

$$\mathbf{W}_M = \kappa \left(\det \mathbf{F} - \frac{1}{\det \mathbf{F}} \right)^2$$

The factor κ was chosen by evaluating numerical experiments. The Cauchy stress tensor τ_M was derived from \mathbf{W}_M by the means described above and also contributed a force to the corner masses.

The described enhancements translated deformations in material stress and resulted in nonlinear spring behavior.

3. Results

Deformation simulations with the hybrid model were conducted upon the patch. The myocardial tension was introduced using the simulations with a force development model. The force development was sampled every 20 ms for the duration of 600 ms. The deformation simulation for each time step was allowed 10'000 iterations to ensure the patch relaxed into a steady state. The patch was fixed at the top layer and no boundary conditions were implemented upon the sides.

The following relative simulation times have been achieved on a single processor SGI 400MHz MIPS R12000. The average volume (average vol.) preservation per voxel and the standard deviation (stddev) is displayed:

model type	time in %	average vol %	stddev
spring model	100	91	0.054
hybrid model	112	98	0.028

The deformation simulation is displayed as a sequence of pictures from side and bottom view (Fig. 7). Numbers indicate time in seconds. The maximal force resulted in maximal deformation at time step 150 milliseconds. The white wireframe denotes relaxed position.

The volume preservation is displayed cutting the patch in the z axis into slices at time step 150 ms. The volume divergence in percent of each voxel in the slice is compared between spring mass system and hybrid model (Fig. 6). The plots indicate that volume preservation has improved. The plots on the right column of Fig. 6 show the regional divergences in a more detailed way.

4. Discussion and conclusion

A hybrid model was presented for simulating cardiac deformation. A spring mass system was enhanced

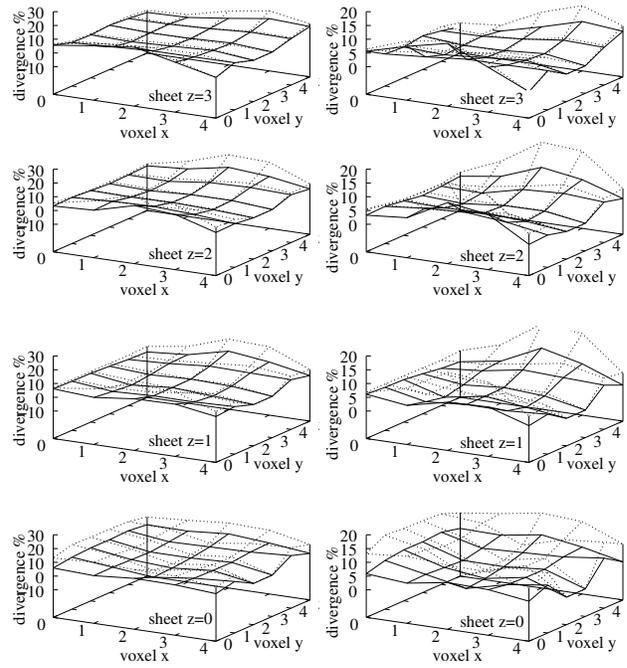


Figure 6. The volume divergence in percent is displayed at time step 150 ms. The patch was parted in slices from top ($z=3$) to bottom ($z=0$) and the volume divergence of each voxel is plotted. The difference between volume divergence of hybrid model (black grid) and spring mass system (dashed grid) are displayed. The right column visualizes a close up in the range of 0 to 20%.

by continuum mechanics based methods to implement nonlinear material properties. Simulations were performed using a patch of a simple ventricle model. Prior calculated force development slopes were utilized to trigger deformation. Simulations showed, that deformations of the patch yielded the expected behavior. The implementation of continuum mechanics resulted in an elimination of spring tuning but led to an 12% increase in computation time. A distinct improvement of isovolumetry using the hybrid model was achieved (Fig. 6). While the improvement in layer $z = 3$ is not too vast, a precise improvement in layer $z = 0$ is noticeable. The rough shape at the borders indicate that further emphasis has to be set on border conditions and patch shape.

5. Future work

Future work will be done concerning computation time aiming for simulations with simple ventricle models and MRI volume data of the left ventricle. Furthermore, isovolumetry will be enhanced and merging of spring and continuum mechanics based methods will be improved.

The border outlines of patches to be simulated has to be investigated. In addition a comparison of simulated and measured velocities of voxels is considered to validate the simulation.

References

- [1] Noble D, Varghese A, Kohl P, Noble P. Improved guinea-pig ventricular cell model incorporating a diadic space, I_{Kr} and I_{Ks} , and length- and tension-dependend processes. *Can J Cardiol* Jan. 1998;14(1):123–134.
- [2] Henriquez CS, Muzikant AL, Smoak CK. Anisotropy, fiber curvature and bath loading effects on activation in thin and thick cardiac tissue preparations: Simulations in a three-dimensional bidomain model. *J Cardiovasc Electrophysiol* May 1996;7(5):424–444.
- [3] Huxley AF. Muscle structures and theories of contraction. *Prog Biophys and biophys Chemistry* 1957;7:255–318.
- [4] Bourguignon D, Cani MP. Controlling anisotropy in mass-spring systems. In *Computer Animation and Simulation '00, Proc. 11th Eurographics Workshop, Interlaken, Switzerland, Springer Computer Science. Springer, Aug. 2000; 113–123.*
- [5] Bathe KJ. *Finite Element Procedures*. Upper Saddle River, New Jersey: Prentice Hall, 1996. ISBN 0-13-301458-4.
- [6] Mohr MB, Blümcke LG, Seemann G, Sachse FB, Dössel O. Volume modeling of myocard deformation using a spring mass system. *LNCS 2673 2003;332–339.*
- [7] Guccione JM. Finite element modeling of ventricular mechanics. In Glass L, Hunter P, McCulloch A (eds.), *Theory of Heart*. Berlin, Heidelberg, New York: Springer. ISBN 3-540-97483-0, 1991; 121–127.
- [8] Sachse FB. *Modeling of the mammalian heart*. Universität Karlsruhe (TH), Institut für Biomedizinische Technik, 2002. Habilitationsschrift.

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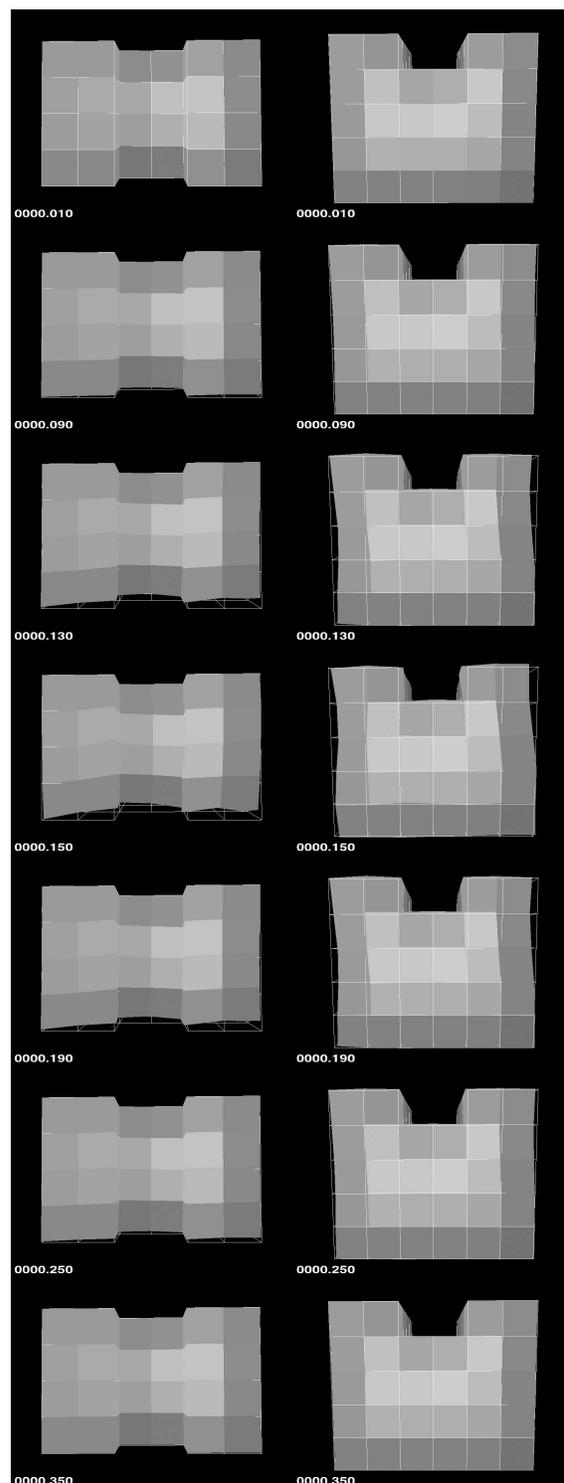


Figure 7. Deformation simulation with the $6 \times 5 \times 4$ voxel patch. The left column shows a front view of the patch. The right column the view from the bottom. The patch was fixed at the top layer. The numbers indicate time in seconds. The white wireframe indicates undeformed state.