

On Utilizing the Strengths of AR and MVDR Methods to Circumvent Their Weaknesses in Application to HRV Analysis

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Abstract

We discuss a procedure that exploits the strengths of autoregressive (AR) and minimum variance distortionless response (MVDR) methods to circumvent their respective weaknesses, in the context of estimating the power-spectral measures from cardiovascular signals. While the AR power spectrum density (PSD) has high resolution, it suffers from a greater degree of variability at low orders and potential spurious peaks at high orders. The MVDR method (of lower resolution) provides the power-spectral measures at lower variability over low filter orders. The weaknesses of the two methods can be circumvented by estimating the powers by low-order MVDR filters at the frequencies estimated by AR PSD. Results of experiments with simulated and real data are supportive.

1. Introduction

The relative role of sympathetic and parasympathetic systems is often assessed through spectral analysis of heart rate variability (HRV) [1], through measures such as the low-frequency (LF) power (LFP~0.1 Hz, attributed primarily to the sympathetic system), the high-frequency (HF) power (HFP~0.2 Hz, attributed to the parasympathetic system) and in particular, the power-ratio, PR: LFP/HFP [2]. A popular approach to computing the preceding spectral measures is based on AR modeling [2],[3],[4]. In spite of its popularity due to high-resolution and the availability of efficient algorithms, the AR approach is more suitable for estimating the frequency-locations than the powers [5],[6]. Being based on the assumption of a model for the data, a violation of the model degrades the performance [7],[8]. Further, selecting the proper order for analyzing actual physical data is difficult [9],[10],[11]. On the other hand, Capon's MVDR method is an optimum power estimator not based on the assumption of a model for the data [12],[13]. While the resolution of the MVDR power spectrum (PS) is lower than that of AR PSD, the former is better behaved with respect to model orders, and affords a lower variability of the spectral measures at lower filter

orders [14],[15].

We consider utilizing the strengths of the two methods to circumvent their respective weaknesses. The basic idea involves using the AR PSD to estimate the frequencies, subsequently used for computing the powers through MVDR filters. This amounts to a resolution-enhancement of the MVDR PS. We show how the variability of the power-spectral measures can be reduced with a proper choice of filter-orders. Note that the concept of using an AR frequency estimator for point-power estimation by MVDR filters was known [6],[16]. However, this was in the context of improved power-estimation in the case of a specific data-model (sinusoids in noise), and did not address statistical variability arising from the use of data-based correlation estimates. Our work is complementary to that of [6].

2. AR and MVDR power spectra

The AR PSD: Modern methods of spectrum estimation attempt to overcome the limitations of the conventional approach based on the Fourier transform (FT) by using a finite parametric model for the data [7]. The most popular is the AR model, due to its success, for example, in speech processing based on the model for speech production [17],[18]. The all-pole structure of the AR model makes it suitable for representing processes with peaky spectra. The AR model is described by:

$$x(n) = -\sum_{k=1}^p a_k x(n-k) + \varepsilon(n), \quad n \geq p \quad (1)$$

where, $\{a_k, k = 0, 1, \dots, p\}$ are the model-coefficients and p is the order. The 'driving process' $\varepsilon(n)$ is a white noise sequence of variance E_p (when the model is appropriate). The PSD associated with the model is:

$$S_{AR}(\omega) = \frac{E_p}{|1 + \sum_{k=1}^p a_k e^{j\omega k}|^2} \quad (2)$$

A major strength of AR PSD is its superior *frequency-resolution*, which stems from the implicit extrapolation of the autocorrelation lags [17],[7]. The linearity of the problem of solving for $\{a_k\}$, the availability of efficient

algorithms for the purpose, and the equivalence of AR and maximum entropy PSDs (in 1-D) have added to the popularity of the method. In addition, autoregressive moving average (ARMA) process can be approximated by an AR model of sufficiently high order [7].

Among the weaknesses is the fact that the AR approach is model-based. Further, it is suitable to estimating the frequency-locations rather than the power [5],[6]. Although the power at a frequency can be estimated as the area under the PSD over an interval, there is no guidance for the choice of the interval. Note that the peak in the AR PSD of a sinusoid in white noise, is proportional to the *square* of the signal power [13],[9] (nevertheless, we found area-based measure to be more tolerant to the noise-strength and model-order than the residue method [19]). Further, variability of the AR spectral measures is higher at lower model-orders [14],[15], and higher orders can create spurious peaks; order-selection is difficult when analyzing real data, and none of the information theoretic criteria is guaranteed to work well, especially on short data [9],[11],[20].

The MVDR power spectrum: An alternative method for estimating the frequencies and powers of dominant components is the MVDR approach [12]. The method treats the problem of estimating the power at a frequency ω as one of designing a unity gain, narrowband filter centered at ω and measuring its output power. The design involves finding a finite impulse response (FIR) filter \mathbf{h} that minimizes the output variance subject to the “Distortionless Response” constraint [13],[9]:

$$\text{Min } P_o(\omega) = \mathbf{h}^T \mathbf{R} \mathbf{h}, \quad \text{Sub : } \mathbf{v}^* \mathbf{h} = 1 \quad (3)$$

where, $\mathbf{v} = [1 \ e^{j\omega} \ e^{j2\omega} \ \dots \ e^{jM\omega}]^T$, M is the *order* of the MVDR filter and \mathbf{R} is the correlation matrix. The power at the output of the optimum filter \mathbf{h} is [13],[9]:

$$P_{MVDR}(\omega) = \frac{1}{\mathbf{v}^* \mathbf{R}^{-1} \mathbf{v}} \quad (4)$$

The MVDR method is not based on the assumption of a model, and directly provides the *power spectrum*. The power estimator involves an implicit narrow-band, data adaptive and frequency-dependent window. The peaks in the MVDR PS are true indicators of the power of sinusoids when the signal-to-noise ratio (SNR) is high [13],[9]. The spurious peaks in the MVDR PS at higher filter orders are subdued relative to those in AR PSD [15]. Finally, the variability of PR estimated by the MVDR approach is lower than those estimated by AR modeling, over lower filter orders [14],[15].

A major weakness of the MVDR power spectrum is its lower frequency resolution compared to that of AR PSD.

3. AR-MVDR procedure

We consider utilizing the strengths of AR (high

frequency-resolution, efficient computation) and MVDR (optimum power, low variability) methods to circumventing their weaknesses (high variability of non-optimum AR power estimates and low resolution of the MVDR PS). Of particular interest is the relation between the AR coefficients and the MVDR PS [21]:

$$P_{MVDR}(\omega) = \frac{1}{\sum_{k=-M}^M \mu(k) e^{j\omega k}} \quad (5)$$

The MVDR coefficients $\mu(k)$ are given by a linearly weighted correlation of the AR coefficients:

$$\begin{aligned} \mu(k) &= \frac{1}{E_M} \sum_{i=0}^{M-k} (M+1-k-2i) a_k^* a_{i+k}^*, \quad k = 0, \dots, M \\ &= \mu^*(-k), \quad k = -1, \dots, -M \end{aligned} \quad (6)$$

where E_M is the AR prediction-error-variance. Eq. (5) and (6) allow efficient computation of the MVDR power: through LD recursions to compute $\{a_k\}$ & E_M , and FFT to compute the PS. The preceding relationship is a result of the specific expression for the power involving \mathbf{R}^{-1} , and does not imply that the MVDR method is model-based.

Based on the known properties of the AR and MVDR methods outlined above, and our recent results on the behavior of the variability of the power-spectral measures estimated by the two methods (also presented in Section 3) a two-step AR-MVDR procedure [15] to estimate the powers of dominant frequency-components follows:

1. Identify the dominant frequencies through AR PSD
2. Estimate the powers at those frequencies by MVDR filters of low order.

Power estimation by lower order (M) MVDR filters, at frequencies determined by possibly higher order (p) AR PSD, seems to be a natural choice, as the frequency-bias of the AR PSD reduces with increasing order [8]. Of note, the *location* of a peak in AR PSD is more robust than the power-estimate, as the *angle* of a pole varies to a lesser degree than its radius [8]. The AR-MVDR procedure would be helpful when the data record is too short for the MVDR spectrum to resolve the frequency components, and/or noisy to estimate the power reliably through AR PSD. For short records, the Burg algorithm can be used to resolve the frequencies, but the “autocorrelation method” [17] is preferable in Step-2 for reduced variability. Consequently, the procedure may be viewed as one that enhances the resolution of the MVDR PS. In this context, we point out a passing remark in [16] that the peaks in the MVDR PS need not be resolved. While high values of p can be used in the case of sinusoids in noise, our experience suggests one to stick to the principle of parsimony in the case of real data.

The concept of using an AR frequency estimator for improved point-power estimation by an MVDR filter was previously known, and used in the specific sinusoid-plus-

noise setting [6],[16]. However, we have arrived at the procedure, not only from the point of using an MVDR power estimator, but also based on the knowledge of the relative behavior of the variability of PR, that exposes a range of (low) filter orders that can be exploited (contrast this with the assumption of the availability of a large number of reliable correlation lags in [6]; also, note that the MVDR estimates of PR suffer from a higher degree of variability at large orders [14],[15]).

4. Results

To motivate the AR-MVDR procedure, we first present the relative behavior of PR, estimated from AR and MVDR methods [14],[15]. The percent values of bias (B_{PR}) and standard deviation (SD_{PR}) of PR, estimated from the two methods, are plotted in Fig. 1, as a function of order. These results were based on 1000 realizations of simulated data (of 60 sec. duration, sampled at 1 Hz), consisting of two unit-amplitude sinusoids (frequencies: 0.1 Hz & 0.2 Hz) of random phase, in additive white Gaussian noise (of strength σ_w). In the AR model-based approach, the power at a frequency f_{peak} was estimated as the area under the PSD over $f_{peak} \pm 0.0125$ Hz. In the MVDR case, the power was read-off at each of the peak-locations. Observe that the values of B_{PR} , and in particular of SD_{PR} , associated with the MVDR method, are lower over a range of lower orders. Although the resolution of the MVDR PS improves with an increase in the order, it is accompanied by an increase in the variability of PR. These graphs reveal the advantage of the MVDR power-estimator within a range of (low) orders, over which the variability of the AR-estimates of PR is significantly higher. On the other hand, the frequencies, at which the powers need to be computed, can be determined by AR PSD of a suitable order.

The AR-MVDR method was implemented with $p=12$ and $M=4$. The resulting values of B_{PR} and SD_{PR} , of 2.4 and 17.2 (in the case of $\sigma_w=0.5$), and 3.3 and 23.1 (in the case of $\sigma_w=1$) demonstrate the efficacy of the procedure. In the case of sinusoids in noise, we found that the performance was not very sensitive to the value of p ($\geq p_{min}$, the order just sufficient to resolve the peaks). For achieving low variability, M should be as low as possible (the MVDR spectral peaks need not be resolved).

We now present the results of application to real data [15]. The plots of SD_{PR} associated with the two methods, estimated from a set of 12, 2-minute segments of HRV data extracted from 6 healthy subjects in supine position on two different days, exhibit a similar property (Fig. 2). The values of “sample” SD_{PR} (SD of the values of PR, estimated from each of the data-segments at the order just sufficient to resolve the LF and HF peaks) associated with the two methods were: 109% (AR) and 97% (MVDR).

The value of $SD_{PR} = 90.6$ associated with the AR-MVDR procedure [15] is supportive.

5. Conclusions

Based on the previously known properties of AR and MVDR methods and our recent results on the properties of the power-ratio estimated by the two methods, the AR-MVDR procedure was proposed. An appropriate choice of filter-orders yields a reduction in the variability of the power-ratio, compared to that estimated by the popular AR approach. Variance-reduction is helpful in data-classification based on the power-ratio. Although the MVDR method by itself affords lower variability, it is limited in frequency-resolution. While resolution is not a major issue in the current application, the resolution-limitation of the MVDR approach can be overcome by the AR-MVDR procedure. Thus, the AR-MVDR procedure circumvents some of the drawbacks of the individual methods.

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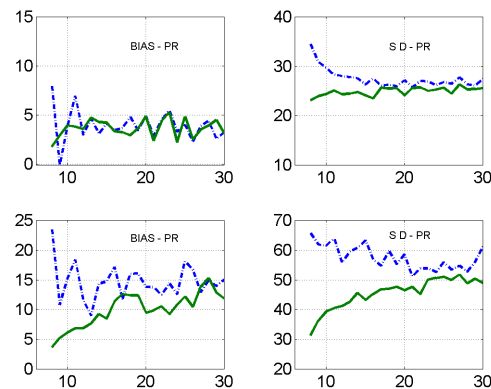


Fig. 1: The values of B_{PR} and SD_{PR} as a function of order. The dotted and solid lines represent the results of AR and MVDR methods, respectively. TOP: $\sigma_w=0.5$, BOTTOM: $\sigma_w=1$.

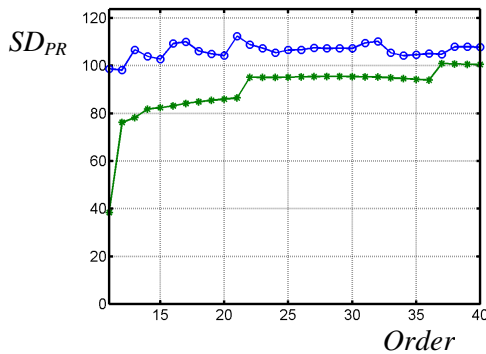


Fig. 2: SD_{PR} as a function of order, estimated from 12 2-minute segments of HRV data. The line with “o” and “*” represent the results of AR and MVDR methods, respectively.

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