Spectrum Estimation and Adaptive Denoising of Electrocardiographic Signals Using Kalman Filters

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Abstract

A method of time–varying parametric spectrum estimation from ECG sequences is presented. Model parameters are estimated recursively using a Kalman algorithm, which extracts the time–varying parameters and state variables of an ECG sequence, as well. We consider the noisy time–sequence generated by nonlinear auto regression, when the observations of the series contain measurement noise in addition to the signal. The spectrum estimates for each time instant then are obtained from the estimated model parameters. Proposed Kalman filter model turns to be adequate for either noise reduction or parameter estimation of processed sequence. Results thus obtained show better performance of Kalman–based filtration algorithm, in the sense of SNR and WDD distortion measurements, in comparison to conventional stationary spectrum estimation.

1. Introduction

The extraction of high resolution ECG signals from noisy measurements, is one of the greatest problems in biomedical signal processing and still remains open until the present. Specifically, the problem that we considered is the estimation of the spectral density of power of an ECG signal from the observation of the same one in a finite time interval. The power spectral density estimation of physiologic signals is performed predominantly using classical techniques based on the Fast Fourier transform (FFT). However, these techniques have some limitations. They require stationarity of the segments studied and have limited frequency resolution [1], [2]. Since ECG signals are nonstationary en nature, these techniques are applied to short overlapping segments which are assumed to be stationary. It imposes a piecewise stationary model on the data and, since local stationarity requires the analysis segments to be short in duration, they have limited time–frequency resolution [3].

Initially the ECG nonstationary time series is treated like a time–varying autoregressive process. Model parameters are estimated recursively using a Kalman algorithm, namely, dual Kalman filter (DKF), which extracts the time–varying parameters and state variables of an ECG sequence, as well.

The adaptive Kalman filter algorithm we propose for instantaneous PSD estimation assumes an underlying autoregressive structure of the data.

We choose an underlying AR(p) model structure because of its intrinsic generality and peak matching capabilities. These are important properties for the analysis of physiologic signals, since we are usually more interested in estimating the frequency at which the formant frequencies (peaks) occur than the valleys.

2. Methods

We propose two kinds of representation for a time–varying series. First, an AR(p) model structure because of its intrinsic generality and peak matching capabilities. These are important properties for the analysis of physiologic signals, since we are usually more interested in estimating the frequency at which the formant frequencies (peaks) occur than the valleys. Second, an ARMA(p,q) process, whose structure is more complete than the AR structure. It has the same properties of peak detection, but it can smooth peaks that may be undesirable, like the ones produced with the additive Gaussian noise.

The general state–space representation of an AR(p) process is as follows

$$\mathbf{x(k+1)} = \mathbf{Ax(k)} + \mathbf{Bu(k)}$$

$$\begin{bmatrix} x(k) \\ x(k-1) \\ \vdots \\ x(k-p+1) \end{bmatrix} = \begin{bmatrix} a_1 & a_2 & \cdots & a_p \\ 1 & 0 & 0 & 0 \\ 0 & \ddots & 0 & \vdots \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x(k-1) \\ x(k-2) \\ \vdots \\ x(k-p) \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} v(k) \quad (1)$$

$$\begin{aligned} y(k) &= \mathbf{Cx(k)} + \mathbf{v(k)} \\ &= [1 \quad 0 \quad \cdots \quad 0]\mathbf{x(k)} + \mathbf{v(k)} \end{aligned} \quad (2)$$

The vector $\mathbf{x}(\mathbf{k})$ which must be estimated is usually referred to as a *state vector*. The current state of the system is defined as the minimal amount of information such that all future behavior of the system can be determined from the future inputs to the system and the current system state. The state is updated through the state transition matrix $\mathbf{A}$, composed of the parameters of the AR(p) polynomial.

For our interest, the output of the model, $y(k)$ is a scalar, and we refer to it as the observation at time k, obtained from the state vector through the observation measurement vector $\mathbf{C}$.

The representation of an ARMA(p, q) process is similar to the AR(p) one. We only have to change the observation measurement vector to the following form

$$\begin{aligned} y(k) &= \mathbf{C}\mathbf{x}(\mathbf{k}) + \mathbf{v}(\mathbf{k}) \\ &= [1 \quad b_1 \quad b_2 \cdots b_q \quad 0 \cdots 0]\mathbf{x}(\mathbf{k}) + \mathbf{v}(\mathbf{k}) \end{aligned} \quad (3)$$

where the b_i's are the parameters of the MA(q) polynomial.

An infinite variety of state–space representations can be found for a linear AR(p) and ARMA(p, q) model by projecting $\mathbf{x}(\mathbf{k})$ on to an alternate basis (*via* a linear transformation). This transformation will of course change the form of the system matrices $\mathbf{A}$, $\mathbf{B}$, and $\mathbf{C}$. The particular form shown here is called the *control canonical representation*, as determined by the special structure of the $\mathbf{A}$ and $\mathbf{B}$ matrices [2].

2.1. Dual Kalman filtering

To estimate the parameters and state of the TVAR process we propose the state space model for an AR(p) in Eq. (2), where $\mathbf{x}(\mathbf{k})$ is the system state and $y(k)$ is an scalar observation, in this case, the observation of the ECG with additive Gaussian noise $v(k)$.

In a dual estimation problem, the objective is to estimate both the state and the parameters of the signal [4], using the model in Eq. (2) and the following model for the parameters dynamics:

$$\mathbf{a}(\mathbf{k+1}) = \mathbf{a}(\mathbf{k}) + \mathbf{r}(\mathbf{k}) \quad (4)$$

$$\mathbf{d}(\mathbf{k}) = \mathbf{a}(\mathbf{k})\mathbf{x}(\mathbf{k}) + \mathbf{e}(\mathbf{k}) \quad (5)$$

where $\mathbf{a}(\mathbf{k})$ is the parameter vector to be estimated, corresponding to the AR(p) process parameters, and $\mathbf{r}(\mathbf{k})$ and $\mathbf{e}(\mathbf{k})$ are interferences to the parameter transition and output equations respectively. The output $\mathbf{d}(\mathbf{k})$ corresponds to a linear observation on $\mathbf{a}(\mathbf{k})$. This approach is more extensively explained in [5].

2.2. The unscented Kalman Filter

2.2.1. Unscented transformation

The unscented transformation (UT) is a method for calculating the statistics of a random variable which undergoes a nonlinear transformation [6]. Consider propagating a random variable $\mathbf{x}$ (dimension L) through a nonlinear function, $\mathbf{y} = \mathbf{f}(\mathbf{x})$. Assume $\mathbf{x}$ has mean $\bar{\mathbf{x}}$ and covariance $\mathbf{P_x}$. To calculate the statistics of $\mathbf{y}$, we form a matrix $\mathcal{X}$ of $2L+1$ sigma vectors $\mathcal{X}_i$ according to the following:

$$\begin{aligned} \mathcal{X}_0 &= \bar{\mathbf{x}} \\ \mathcal{X}_i &= \bar{\mathbf{x}} + \left(\sqrt{(L+\lambda)\mathbf{P_x}}\right)_i, \quad i = 1, ..., L \\ \mathcal{X}_i &= \bar{\mathbf{x}} - \left(\sqrt{(L+\lambda)\mathbf{P_x}}\right)_{i-L}, \quad i = L+1, ..., 2L \end{aligned} \quad (6)$$

where $\lambda = \alpha^2(L+k) - L$ is a scaling parameter. The constant α determines the spread of the sigma points around $\bar{\mathbf{x}}$, and is usually set to a small positive value (e.g., $1 \leq \alpha \leq 10^{-4}$). The constant k is a secondary scaling parameter, which is usually set to $3 - L$, and β is used to incorporate prior knowledge of the distribution of $\mathbf{x}$ (for Gaussian distributions, $\beta = 2$ is optimal). $\left(\sqrt{(L+\lambda)\mathbf{P_x}}\right)_i$ is the ith column of the matrix square root (e.g., lower–triangular Cholesky factorization). These sigma vectors are propagated through the nonlinear function

$$\mathcal{Y}_i = f(\mathcal{X}_i), \quad i = 0, ..., 2L \quad (7)$$

and the mean and covariance for $\mathbf{y}$ are approximated using a weighted sample mean and covariance of the posterior sigma points,

$$\bar{\mathbf{y}} \approx \sum_{i=0}^{2L} W_i^{(m)} \mathcal{Y}_i \quad (8)$$

$$\mathbf{P_y} \approx \sum_{i=0}^{2L} W_i^{(c)} (\mathcal{Y}_i - \bar{\mathbf{y}})(\mathcal{Y}_i - \bar{\mathbf{y}})^\top \quad (9)$$

with weights W_i given by

$$\begin{aligned} W_0^{(m)} &= \frac{\lambda}{L+\lambda} \\ W_0^{(c)} &= \frac{\lambda}{L+\lambda} + 1 - \alpha^2 + \beta \\ W_i^{(m)} &= W_i^{(c)} = \frac{1}{2(L+\lambda)}, \quad i = 1, ..., 2L \end{aligned} \quad (10)$$

2.2.2. Implementation of the unscented Kalman filter

The unscented Kalman filter (UKF) is a straightforward extension of the UT to the recursive estimation in,

$$\hat{\mathbf{x}}_k = (\textit{prediction of } \mathbf{x_k}) + \mathcal{K}_\mathbf{k}[\mathbf{y_k} - (\textit{prediction of } \mathbf{y_k})] \quad (11)$$

where the state random variable is redefined as the concatenation of the original state and noise variables: $\mathbf{x}_\mathbf{k}^\mathbf{a} = [\mathbf{x}_\mathbf{k}^\top \quad \mathbf{v}_\mathbf{k}^\top \quad \mathbf{n}_\mathbf{k}^\top]^\top$. The UT sigma point selection scheme, Eq. (7), is applied to this new augmented state RV to calculate the corresponding sigma matrix, $\mathbf{X}_k^a$ [5].

2.3. Time–varying spectrum estimation

The time–varying spectrum estimate is obtained from the momentary AR(p) parameter estimates $\hat{a}_t^k$ as

$$P_t(f) = \frac{\hat{\sigma}_e^2/f_s}{|1 + \sum_{k=1}^{p} \hat{a}_t^k e^{-j2\pi kf/f_s}|^2}, \tag{12}$$

where f_s is the sampling frequency, $\hat{a}_t^k$ is the AR(p) parameter estimate at time t and $\hat{\sigma}_e^2$ is the variance of the estimated observation error process. Note that Eq. (12) is a continuous function of frequency and can, thus, be evaluated at any desired frequencies up to the Nyquist frequency $f_s/2$. However, the frequency resolution is naturally not infinite, but is determined by the underlying parameter model, i.e., the model structure and model order. When compared to classical FFT–based spectrum estimation methods, the resolution of parametric methods is higher due to the implicit extrapolation of the autocorrelation sequence.

The characteristics of the Kalman smoother spectrum depend strongly on the order of the AR(p) model. As a rule of thumb it can be said that a smaller model order results in a smoother spectrum and a selection of too high–order model can produce spurious peaks in the spectrum, but in any case the order should be at least twice the number of expected peaks in the spectrum.

2.4. The weighted diagnostic distortion (*WDD*) measure

For every beat of the original signal and for the reconstructed signal, a vector of diagnostic features is defined [7]:

$$\begin{aligned} \beta^\top &= [\beta_1 \quad \beta_2 \quad \cdots \quad \beta_p], \\ \hat{\beta}^\top &= \left[\hat{\beta}_1 \quad \hat{\beta}_2 \quad \cdots \quad \hat{\beta}_p\right], \end{aligned} \tag{13}$$

where β_p is the number of features in the vector, $p = 15$ is used in this work. The diagnostic parameters ($\beta_i,\ i = 1, 2, \ldots, p$) were chosen to be: QRS_{dur}, QRS^+_{amp}, QRS^-_{amp}, $area^+_{QRS}$, $area^-_{QRS}$, $area_T$, $ST_{elevation}$, ST_{slope}, PR_{int}, T_{amp}, QT_{int}, QTp_{int}, P_{amp}, P_{dur}, RR_{int}. The *WDD* between these two vectors is

$$W_{DD}(\beta, \hat{\beta}) = \Delta\beta^\top \cdot \frac{\Lambda}{tr[\Lambda]} \cdot \Delta\beta \times 100 \tag{14}$$

where $\Delta\beta$ is the normalized difference vector

$$\beta^\top = [\Delta\beta_1 \quad \Delta\beta_2 \quad \cdots \quad \Delta\beta_p] \tag{15}$$

and Λ is a diagonal matrix of weights, defined in $\Lambda = diag[\lambda_i]$, $\lambda_i > 0,\ i = 1, 2, \ldots, p$. Every scalar in this vector gives the distance between the original signal feature and the reconstructed signal feature. For the duration features and the amplitude features, the distance is defined as

$$\Delta\beta_i = \frac{\left|\beta_i - \hat{\beta}_i\right|}{\max\left\{|\beta_i|, \left|\hat{\beta}_i\right|\right\}}. \tag{16}$$

3. Results

We tested the filtering performance of the unscented Kalman filter in synthetic ECG signals [8], [9], with additive white Gaussian noise and $SNR = 3\,dB$, using the MSE measurement and the WDD measurement. In the same way we have tested the performance of the filter, using different orders of the AR(p) process. Namely, we have used the following orders: $p = (16,\ 22,\ 26,\ 30)$.

The results of these tests for MSE and WDD are summarized in Figs. 1 and 2.

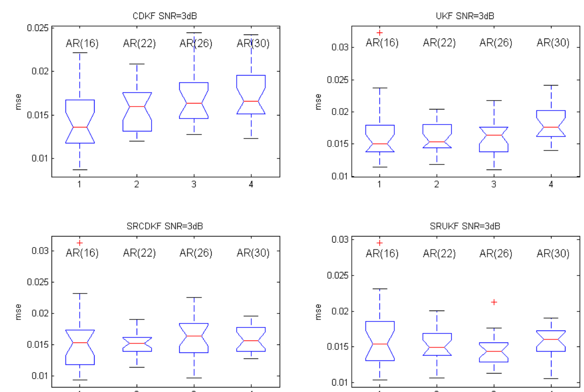


Figure 1. MSE of filtered signal with $SNR = 3\,dB$.

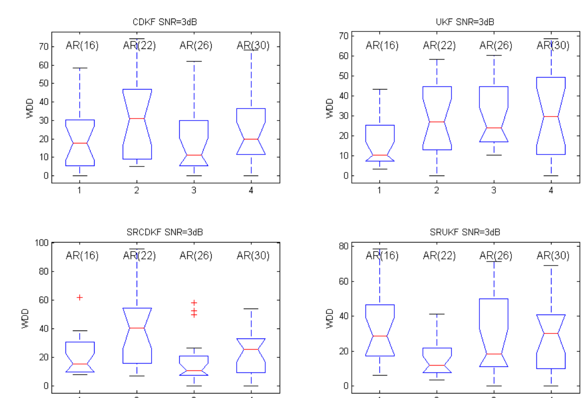
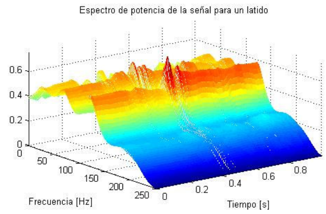
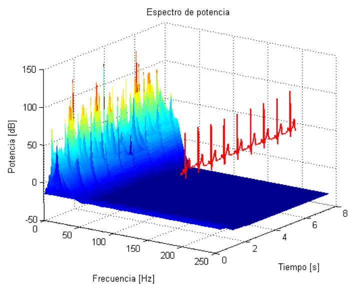


Figure 2. WDD of filtered signal with $SNR = 3\,dB$.



(a)PSD of the signal estimated with DKF



(b)PSD of the clean signal

Figure 3. PSD of ECG signals.

4. Discussion and conclusions

According to the boxplots of the MSE, we can see that it's not very sensitive to the order of the AR(p) model, because the improvement of the MSE is not significative while the order of the AR(p) model increases. When the SNR of the signal is low, an increment of the order makes the error criterion to be bigger, in this way, higher orders of the model doesn't imply better performance of the algorithm. This due to the overfitting caused because the order of the model is higher than the order needed to model correctly the signal to be filtered.

On the other hand, the WDD measure reflects that the algorithm induces a lot of distortion on the signal, and gives no information about the dependence of the order of AR(p) model, because when the model order increases there is not tendency in the error criterion.

The PSD gives important information about what is going on, because unlike the spectrum of the original signal, that has the power distribution between 0 and 100 Hz, while the spectrum of the estimated signal has its power distributed along all frequencies.

We have used the Kalman filter to estimate both the signal and the parameters of the process that generates such signal with average results. The filtered signal with the MSE measure seems to be adequate, but the WDD measure demonstrates that the signal is too distorted. The PSD also reflects that strange behavior because, as it was seen in Fig. 3, the spectrum is distributed along all frequencies.

This behavior can be improved using a more deterministic kind of noise like colored noise or (50–60) Hz noise.

Anyway, the Kalman filter is very useful to model and filter those ECG signals contaminated by white Gaussian noise. With the Kalman Filter we could estimate both parameter and state of the clean signal without noise, and, in this way, we could filter the contaminated signal. Since Kalman filter is a time variant approach, we can use it to filter non–stationary signals, like the ECG that are difficult to filter with conventional stationary methods.

Even so the MSE is a good criterion to compute the level of noise of any signal, for the ECG signals is better to use an advanced method to measure the quality of the filter that has been implemented. With the WDD measure, we can to compare distortion between the original ECG signal without noise and the filtered signal. Because of the WDD is based on diagnostic features, the WDD contains direct diagnostic information and is more useful than other methods.

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