# Dense Motion Estimation of the Heart Based on Cumulants

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## Abstract

*Mutual Information (MI) has been extensively studied as similarity measure for the registration of medical images, and it has been found to be especially robust for multimodal image registration. However, MI estimators are known i) to have a very high variance and ii) to be computationally costly. In order to overcome these drawbacks, we propose a new similarity measure based on the sum of squared cumulants. In addition, our measure can be easily derivated with respect to registration parameters leading to an optimization with a simple gradient rule. Such a scheme is presented for a non-rigid registration and its performance is studied through computer results in the context of cardiac multislice computed tomography.*

## 1. Introduction

MultiSlice Computed Tomography (MSCT) imaging offers advantages to study both cardiac anatomy and function. Cardiac function assessment from 3D image sequences has been greatly improved by the recent technical developments: 3D echography, cine-MRI and MSCT providing 3D dynamic images in a single exam. But these 3D images need the development of adapted tools to extract cardiac movement. Motion estimation problems can be viewed as registration problems. Many automatic medical image registration methods have been proposed (see [1] for a bibliographical survey). Among them, the techniques based on Mutual Information (MI) [2] have become standards of processing in the context of multidimensional non-rigid and multimodal medical image registration [3] since their first use in such applications [4, 5].

The computation of the MI requires the estimation of marginal and joint Probability Density Functions (PDF's) as we will see in section 2. Consistent kernel estimators like Parzen estimators can be used. But in practical contexts, the integral computation of such estimators is time-consuming and the estimation of PDF's from a finite set of data generally implies a high variance estimate. A simple way of avoiding these drawbacks was proposed two decades ago in Independent Component Analysis (ICA). It is noteworthy that the ICA scheme aims at identifying the statistical independent components of a noisy static mixture [6]. Although the MI was shown to be an appropriate independent measure to perform ICA, cumulants appeared to be useful statistical tools easier to handle [7]. Cumulants allow to measure the statistical dependence of two random variables, vanishing if the two components are independent.

As a result, in order to overcome the drawbacks of the MI estimators, we propose in this paper a new similarity measure based on cumulants. It can be viewed as a generalization of the classical Mean Square Error (MSE) to higher order statistics. Moreover, measure can be easily derived with respect to registration parameters allowing for an optimization with a simple gradient rule. Such a scheme is presented for a non-rigid registration of MSCT cardiac images and its performance is studied through computer results showing its good behavior.

## 2. Toward a novel dependence measure

The major components of a registration framework are basically the feature space (the characteristics of the images taken in account), the similarity measure used to compare these characteristics, the type of transformation we consider, and the chosen optimization method. In this section, we focus on the new similarity measure proposed in this paper to register images.

In information theory, the MI of two random variables $x$ and $y$ gives a measure of the statistical dependence of both variables. This Kullback-Leibler divergence can be expressed as a function of the marginal and joint entropies of $x$ and $y$:

$$MI(x, y) = H(x) + H(y) - H(x, y) \quad (1)$$

In fact, Shannon entropy [8] of $x$, $H(x)$, is a measure of the average or expected information content of an event described by the random variable $x$, given by:

$$H(x) = -\sum_{u} p_x(u) \log(p_x(u)) \quad (2)$$

where $p_x$ is the marginal probability distribution of $x$. The joint entropy $H(x,y)$, measuring the dispersion of the joint PDF, $p_{x,y}$, of the couple $(x,y)$, is defined by:

$$H(x,y) = -\sum_{u,v} p_{x,y}(u,v)\log(p_{x,y}(u,v)) \qquad (3)$$

As shown in equations (1)-(3), in practice the estimation of the MI requires the estimation of the marginal and joint PDF's of the couple $(x,y)$. Recall also that it is possible to derive a metric (or distance function), $D_{\mathrm{MI}}$, from the MI, say a function which defines a distance between variables of the set of the random variables with values in R:

$$\begin{array}{rcl} D_{\mathrm{MI}}(x,y) & = & H(x,y) - MI(x,y) \\ & = & 2H(x,y) - H(x) - H(y) \end{array} \qquad (4)$$

It is noteworthy that a function $D : F \times F \to \mathrm{R}$ is a metric on a set $F$ if, for all $x$, $y$ and $z$ in $F$, we get:

A1. $D(x,y) \geq 0$ with equality if and only if $x = y$;
A2. $D(x,y) = D(y,x)$;
A3. $D(x,y) \leq D(x,z) + D(z,y)$;

The first axiom ensures the positive definiteness of the $D$ function. Note that if $F$ denotes the set or a subset of the random variables, the equality between $x$ and $y$ has to be observed almost surely, say with a probability equal to one. The second axiom means that $D$ is symmetric and the third axiom refers to the triangle inequality. In addition, we have $D_{\mathrm{MI}}(x,y) \leq H(x,y)$ for all couples $(x,y)$ of variables, which implies that the values $D_{\mathrm{MI}}(x,y)/H(x,y)$ will be always upper bounded by one.

Another way to quantify the amount by which a random variable $x$ differs from another one $y$ consists in computing the MSE between both variables, say:

$$\begin{array}{rcl} MSE(x,y) & = & \mathsf{E}[(x-y)^2] \\ & = & \mathsf{E}[x^2] + \mathsf{E}[y^2] - 2\mathsf{E}[xy] \end{array} \qquad (5)$$

where $\mathsf{E}[x]$ denotes the mathematical expectation of $x$. Such a measure only involves the marginal and joint second moments of the couple $(x,y)$, which are easily estimated in practical contexts under some mild conditions. In fact, the MSE is also a metric but this time on the subset of the second order random variables in comparison with the $D_{\mathrm{MI}}$ distance.

Now, let $\Phi_x(u) = \mathsf{E}[\exp(\mathrm{iux})]$ be the first characteristic function of the random variable $x$. Since $\Phi_x(0) = 1$ and $\Phi_x$ is continuous, then there exists an open neighborhood of the origin, in which $\Psi_x(u) = \log(\Phi_x(u))$ can be defined. Function $\Psi_x$ is called the second characteristic function of $x$. The coefficients of the Taylor expansion $\Psi_x$ in the neighborhood of the origin allow to define special statistical quantities, called *cumulants* [9]. Cumulants are also named semi-invariants in statistics and can be explicitly related to moments as illustrated below [10]:

$$\begin{array}{rcl} \mathcal{C}_x^{(2)} & = & \mathsf{E}[x^2] \;\; = \;\; \mathrm{Var(x)} \\ \mathcal{C}_x^{(3)} & = & \mathsf{E}[x^3] \\ \mathcal{C}_x^{(4)} & = & \mathsf{E}[x^4] - 3\mathsf{E}[x^2] \end{array} \qquad (6)$$

where $\mathcal{C}_x^{(2)}$, $\mathcal{C}_x^{(3)}$ and $\mathcal{C}_x^{(4)}$ are the Second Order (SO), Third Order (TO) and Fourth Order (FO) marginal cumulants of the zero-mean, unit-variance variable $x$. Then we propose the $\Psi$ measure given by:

$$\Psi_{\alpha,\beta,\gamma}(x,y) = \alpha\,\mathcal{C}_{x-y}^{(2)} + \beta\left(\mathcal{C}_{x-y}^{(3)}\right)^2 + \gamma\left(\mathcal{C}_{x-y}^{(4)}\right)^2 \qquad (7)$$

where $\alpha$, $\beta$ and $\gamma$ are strictly positive real numbers. It may be asked whether the $\Psi$ measure is well a metric on the set of the random variables with finite SO, TO and FO cumulants. The axioms A1 and A2 are well-satisfied. Indeed, from (6) and (7), the $\Psi$ measure is always positive. Besides, even if some variables $z = x - y$ may have zero TO and FO cumulants for $z \neq 0$ such as the Gaussian one, their SO cumulant will be always non-zero. As far as the third axiom is concerned, there is no trivial result about it using our measure. This will be studied in a forthcoming work.

## 3. A cumulant-based registration scheme

In the following, $I_R$ and $I_F$ will denote the reference image and the floating image on which a transformation $\phi$ will be applied to perform the registration, respectively. The chosen transformation is a Free-Form Deformation (FFD) model based on B-splines, which is a powerful tool for modeling 3D deformable objects. It warps an image by moving an underlying set of control points distributed over a regular grid. The displacement of a point $\boldsymbol{v}$ of the image can be written as a linear combination of B-spline functions $\beta^{(k)}$, weighted by the parameters $\boldsymbol{\xi}^{(k)}$ in the neighborhood $\mathcal{K}(\boldsymbol{v})$ of this point:

$$\phi(\boldsymbol{v},\xi) \;=\; \boldsymbol{v} + \sum_{k\in\mathcal{K}(\boldsymbol{v})} \boldsymbol{\xi}^{(k)}\,\beta^k(\boldsymbol{v}) \qquad (8)$$

Interesting properties of the B-splines for our study are the compacity of their support, derivability and separability in each dimension. The aim of the registration process is to align the pixels $I_F(\phi(\boldsymbol{v},\boldsymbol{\xi}))$ of the transformed floating image with the pixels $I_R(\boldsymbol{v})$ of the reference image. The vector $\boldsymbol{\xi}$ describes the B-spline coefficients to be determined. Then, the registration can be formulated as a minimization problem:

$$\widehat{\boldsymbol{\xi}} = \arg\min_{\boldsymbol{\xi}} \Psi_{\alpha,\beta,\gamma}(I_R, I_F(\phi(.,\boldsymbol{\xi}))) \qquad (9)$$

A gradient descent is thus used in order to find the $\widehat{\boldsymbol{\xi}}$ parameter which minimizes the cumulant-based similarity metric $\Psi_{\alpha,\beta,\gamma}$. At iteration $it$, we take the actual estimate $\boldsymbol{\xi}(it)$ of the vector parameter $\boldsymbol{\xi}$ and calculate an update $\boldsymbol{\xi}(it+1)$ by using the following rule:

$$\boldsymbol{\xi}(it+1) = \boldsymbol{\xi}(it) - \mu(it)\, \nabla\Psi_{\alpha,\beta,\gamma}(I_R, I_F(\phi(., \boldsymbol{\xi}(it)))) \tag{10}$$

This iterative scheme is performed until convergence, where $\mu(it)$ is the step size of the gradient descent which requires to be adjusted at each iteration $it$.

## 4. Computer results

The aim of this section is to analyze the behavior of the cumulant-based registration scheme in comparison with the MSE-based method through computer simulations. In our experiment we use two dimensional MSCT slices of $(200 \times 200)$ pixels. We present experiments in a controlled environment, allowing an exact evaluation of the registration accuracy. A $(200 \times 200)$ image (Fig. 2(b)) is extracted from an original MSCT slice (Fig. 2(a)) in order to form the floating image. Next, we non-rigidly transform the floating image using a $(10 \times 10)$ deformation grid (Fig. 2(e)) of B-spline control points to obtain the reference image (Fig. 2(c)). Then, we try to find back the deformation using the cumulant-based and MSE-based optimization procedures from the floating and reference images. To compare the performance of both approaches, we compute the distance between the vector $\boldsymbol{\xi}$ of B-spline coefficients and its estimate $\widehat{\boldsymbol{\xi}}$ given by:

$$D(\boldsymbol{\xi}, \widehat{\boldsymbol{\xi}}) = \frac{||\boldsymbol{\xi} - \widehat{\boldsymbol{\xi}}||}{\mathrm{length}(\boldsymbol{\xi})} \tag{11}$$

where $||\boldsymbol{h}||$ denotes the Euclidean norm of vector $\boldsymbol{h}$.

Figure 1 shows the distance (11) at the output of both approaches as a function of the number of iterations of the gradient rule (10). The convergence to zero can be observed for both methods with a slightly smoother curve for our cumulant-based approach. In addition, the registered image obtained using the cumulant-based similarity measure is shown in figure 2(d). It appears to be very close to the reference image as shown in figure 3 for which the absolute value between both images was computed.

## 5. Conclusion

Recently, MI and its normalized versions have emerged as effective similarity measures for image registration. However, MI estimators are known i) to have a very high variance and ii) to be computationally costly. Conversely, the MSE similarity measure may appear to be very attractive. Nevertheless, it uses only the SO statistical information of the data, which may be restrictive when the considered images are not Gaussian.

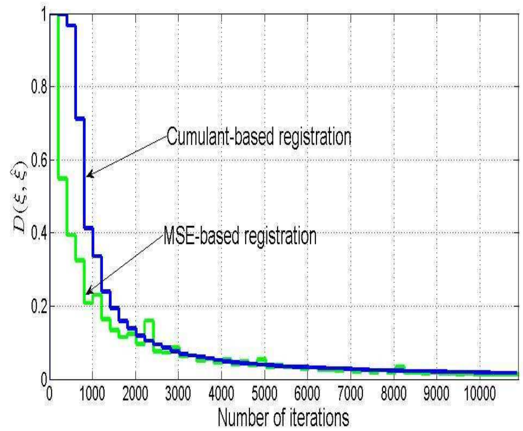


Figure 1. Non-rigid image registration using $10 * 10$ grid of B-spline control points to parametrize the deformation field for $(200 \times 200)$ images.

As a result, we proposed a new similarity measure based on $q$-th ($q \leq 4$) order cumulants, which generalizes the MSE measure to higher order statistics. Our measure then can use more statistical information of the data for an acceptable variance of estimation. In addition, it can be easily derivated with respect to registration parameters allowing for an optimization with a simple gradient rule.

So, we developed a fully automatic and intensity-based registration algorithm with a parametric model of the deformation such as the B-spline model, which is computationally more efficient than other alternatives. Such a scheme is evaluated for the non-rigid registration of MSCT cardiac slices and its performance is compared with the MSE-based procedure. Computer results show the good behavior of our cumulant-based registration technique.

Forthcoming works will extend the proposed cumulant-based registration algorithm to a double multiresolution strategy (for both images and B-spline transformation grid), which will allow us to perform a time efficient registration of 3-dimensional images. Besides, in this paper, we did not prove that our measure statisfies the triangle inequality, even if our measure gives good registration results. Such a study will be given in a longer paper.

## References


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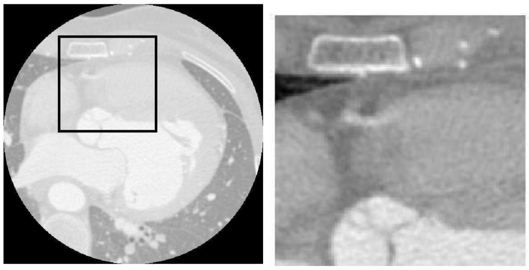
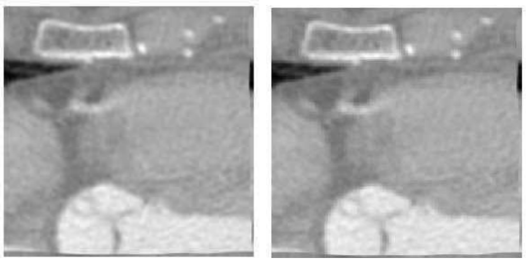
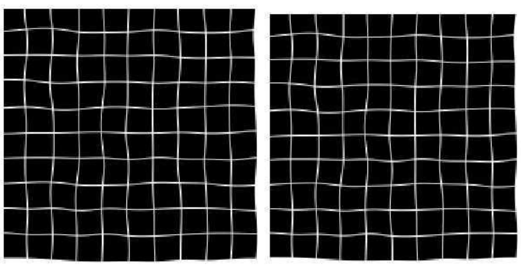


(a)MSCT slice (b)Floating image



(c)Reference image (d)Registered image



(e)Deformation grid (f)Estimated grid

Figure 2. Example of a non-rigid registration for MSCT slices. The registered image (d) nicely resembles the reference image (c) using the B-spline transformation. The original (e) and estimated deformation (f) grids are transformed respectively according to the original and estimated deformation field.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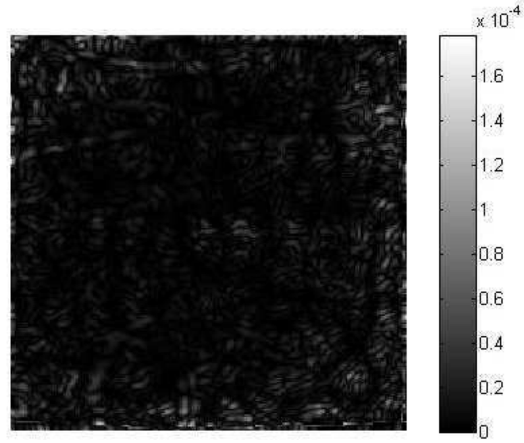


Figure 3. Absolute value of the difference between the reference image (Fig. 2(c)) and the registered image (Fig. 2(d))


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