Relationship between Fractal Dimension and Power-Law Exponent of Heart Rate Variability in Normal and Heart Failure Subjects

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Abstract

Among the plethora of indices that can describe the fractal-like behaviour of heart rate variability (HRV), the fractal dimension (FD) and the power-law exponent (β) have gained wide acceptance. Since HRV is generally modelled with fractional Brownian motion (fBm), the linear scaling relationship between β and FD, valid for fBm, is often applied to HRV series to derive one index from the other. In this paper the relationship between β and FD is calculated in normal (NR) and heart failure (HF) HRV series. Results revealed that a linear dependence between β and FD can be found only when the slope of the spectral density is calculated over the whole spectrum instead of considering more widespread very low frequency ranges. Moreover, the relationship is slightly different from that characterizing fBm and is not unique for the two categories of subjects. The common practice of estimating β from FD for HRV applying the theoretical relationship should be reconsidered.

1. Introduction

The analysis of the heart rate variability (HRV) is a well-recognized tool in the study of the autonomic control of the heart [1]. Beside traditional linear parameters, defined either in time or in frequency domain and able to provide valuable diagnostic and prognostic information on cardiovascular diseases, several nonlinear indices, based on chaos theory and fractal analysis, have been more recently introduced [2-6]. However, although nonlinear indices could be powerful tools in medical assessment, clinicians are not yet favourably disposed towards them. In fact, while the physiological meaning of HRV linear indices is clear, nonlinear techniques appear to be mere mathematical algorithms. This work adds to many other extensive studies recently carried out in order to promote the clinical relevance of the proposed nonlinear methodologies, also trying to clarify the relationships existing among them [2,3].

Since the RR series follows power-law spectrum in frequency and fractal-like pattern in time, HRV is considered a self-affine process. Two of the most widespread indices that mathematically characterize self-affinity are the β exponent of the $1/f^{\beta}$ power spectrum and the fractal dimension FD [7,8]. Since both indices measure the same feature of HRV, a certain relationship is expected to exist between them. In order to correctly characterize HRV by either β or FD, a model that accurately describes the heart rate variability is necessary.

Normal HRV shows fluctuations similar to those displayed by long-memory stochastic processes like fractional Gaussian noise (fGn) or fractional Brownian motion (fBm) [9]. In particular, being β values of HRV largely included between 1 and 2, therefore in the fBm domain, fractional Brownian motion is usually considered as a suitable mathematical model for HRV series. For a fBm stochastic process the fractal dimension is linked to the power-law exponent by the following linear relationship [10]:

$$FD = \frac{5-\beta}{2} \quad (1)$$

where β is included in the 1-3 interval and FD spans the range 1-2. The scaling relationship (1) has been widely exploited by researchers to obtain one nonlinear index from the other [2,11].

In this study the relationship between the power-law exponent and the fractal dimension of HRV series was calculated in normal and heart failure subjects. Aim of the work was to verify if the theoretical relationship (1), valid for fBm, holds also for HRV series, focusing on the possible differences between the two populations.

2. Materials and methods

2.1. Subjects

The study was performed on 50 normal subjects (mean age 42) and 50 stable chronic heart failure patients in sinus rhythm (mean age 51, LVEF 24%).

2.2. HRV series

The HRV series were derived from 24-hours Holter recordings performed using a two-channel device. Each beat was labeled as normal or aberrant by the software and successively checked by an experienced physician. Identified RR time series were then pre-processed, according to the criteria listed in the study of Maestri et al. [3], in order to correct RR intervals associated with ectopic beats, arrhythmic events and artefacts that may alter the characteristics of HRV [1]. Edited RR time series, resampled at 2Hz, composed the two groups of 50 NR-HRV and 50 HF-HRV analysed in the study.

2.3. Estimation of the fractal dimension

The fractal dimension was estimated with Higuchi's algorithm, developed for the analysis of irregular time series directly in the time domain [12]. Let $x(1), x(2), \ldots, x(N)$ be the time series under investigation. Construct k new time series as follows:

$$x_k^m = \left\{x(m), x(m+k), x(m+2k), \ldots, x(m+\left\lfloor\frac{N-m}{k}\right\rfloor k)\right\}$$
$$m = 1,2,\ldots,k \qquad m,k \in \mathbb{N}_0$$

where m represents the initial time and k, ranging from 1 to k_{max}, indicates the time delay. In this work $k_{max} = 6$. The symbol $\lfloor a \rfloor$ denotes the integer part of a. For each x_k^m constructed series the length $L_m(k)$ is calculated as:

$$L_m(k) = \left[\left(\sum_{i=1}^{\left\lfloor\frac{N-m}{k}\right\rfloor} |x(m+ik) - x(m+(i-1)k)|\right) \cdot \frac{N-1}{\left\lfloor\frac{N-m}{k}\right\rfloor k}\right] \cdot \frac{1}{k}$$

where the term

$$\frac{N-1}{\left\lfloor\frac{N-m}{k}\right\rfloor k}$$

is a normalization factor. The length $L(k)$ is computed for all time series having the same time delay k as the average of the k lengths $L_m(k)$ for $m = 1,2,\ldots,k$. If $L(k) \propto k^{-FD}$, the time series x is fractal with dimension FD. Thus, if $L(k)$ is plotted against $1/k$ on a double logarithmic scale, the slope of the straight line fitting the data represents a good estimate of the FD value.

The FD values were calculated on 1-hour consecutive HRV intervals in order to detect short-term variations insight the 24-hours.

2.4. Calculation of the power-law exponent

The power-law exponent was calculated from the power spectrum density (PSD) estimated by periodogram method after Hamming windowing. The β index represents the slope of the linear best-fit of the PSD on a double logarithmic scale. For the HRV series the slope is usually calculated in narrow frequency bands including very low frequency components. In this work the β index was calculated in three traditional frequency bands, as well as in the whole spectrum.

The first index, hereafter called β_1, was defined in the 0<f<0.005Hz frequency range [2] while the second one, β_2, was calculated for 0<f<0.01Hz [3,6]. The third value, β_3, was obtained by regressing the power spectrum for frequencies in the range 0.02<f<0.04Hz [4]. The last index, β_4, was defined in the whole spectrum (0<f<0.45Hz). The frequency ranges considered for the power-law exponents evaluation are summarized in Table 1.

The β_i indices were calculated on the same time intervals used for the estimation of the fractal dimension.

Table 1. Frequency ranges in which the four different power-law exponents were calculated.

	frequency range [Hz]
β_1	0< f <0.005
β_2	0< f <0.01
β_3	0.02< f <0.04
β_4	0< f <0.45

2.5. FD-β relationships

For each RR time series, FD was plotted against β_i. Since a linear dependence was expected to exist between FD and β_i, the Pearson's correlation coefficient r_i was calculated for each (β_i, FD) pair. Slope and intercept of the linear regression were then calculated only for the highest correlated (β_i, FD) pairs and averaged over the 50 NR-HRV and the 50 HF-HRV separately. The mean relationships were finally compared with each other and with the theoretical one valid for the fBm processes (1). A t-test was performed to assess statistically significant differences in the relationships between the two populations.

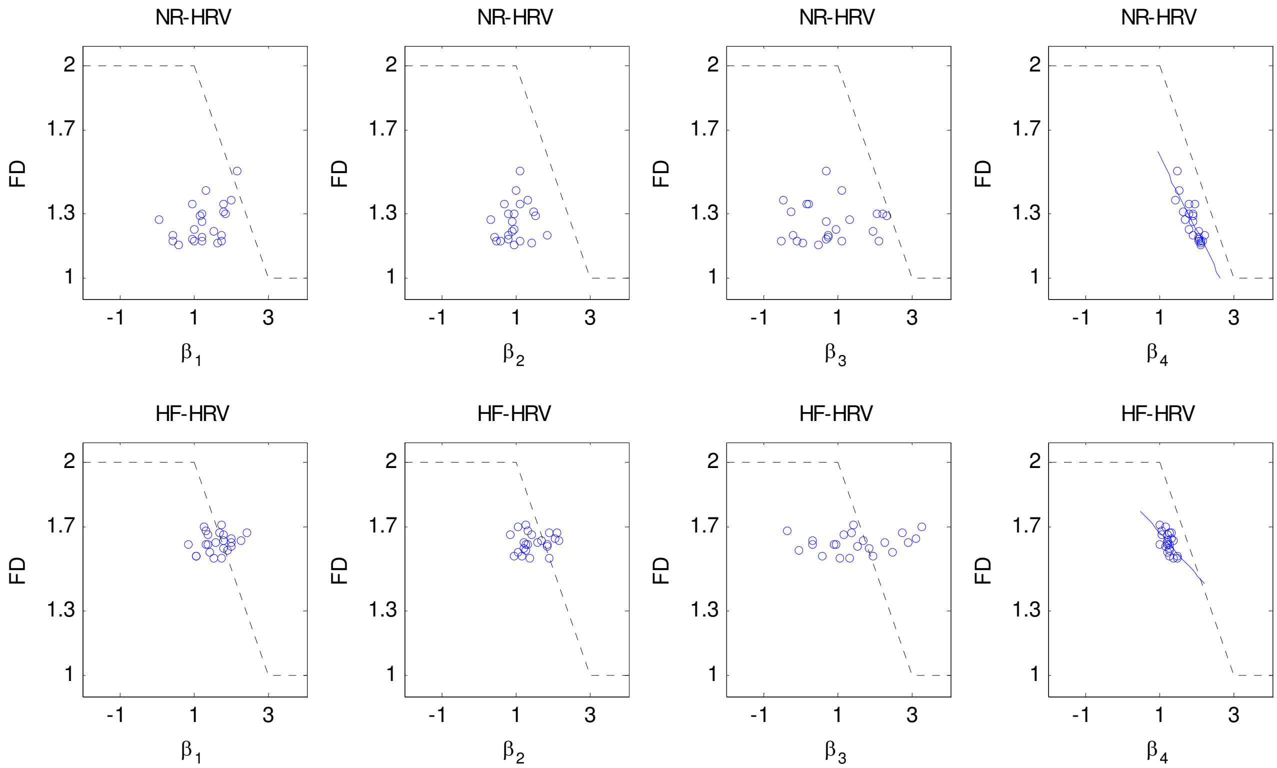

Figure 1. Fractal dimension FD plotted against β_i for the HRV series of a normal subject (top row) and of a heart failure patient (bottom row). The dashed black line, for β in the range 1-3, represents the linear relationship valid for the fBm processes. This relationship is not valid neither for β <1 (fGn processes) nor for β >3 at which it saturates to a FD value of 2 and 1, respectively. The solid blue line in the last graph of each row represents the calculated relationship between β_4 and FD.

3. Results

As can be seen in Figure 1, the β_i indices showed different values depending on the frequency range in which they were calculated. Furthermore, the power spectrum density of HRV series didn't show a perfect $1/f^{\beta}$ trend, even for small frequency bands, suggesting no true self-affine behaviour.

Moreover, a linear relationship between β and FD was detectable only in the case of β_4, i.e. when the slope of the PSD was calculated in the whole frequency range. In the other three cases, in which the exponents were calculated in more widespread low-frequency ranges, β and FD were almost independent. These results are confirmed by the correlation analysis. The highest Pearson's correlation coefficients, in absolute value, were those related to the (β_4, FD) pairs (r_4). The mean correlation was equal to 72% for NR-HRV and to 58% for HF-HRV. The mean p-value for testing the hypothesis of no correlation was 0.025 in both groups, suggesting that the correlation values can be considered significant.

The linear relationships were finally calculated only for the (β_4, FD) pairs. Their mean coefficients, obtained by averaging the 50 relationships of each group, showed that the experimental relationship was not the same for NR-HRV and HF-HRV series (Table 2). In particular, the slope values were significantly different ($p<<0.001$) for the two populations.

Furthermore, both relationships slightly deviated from the theoretical one valid for fBm (1) in which slope and intercept are equal to -0.5 and 2.5, respectively.

Table 2. Mean coefficients ($\mu\pm\sigma$) of the linear relationship between β_4 and FD for normal and heart failure HRV series.

	NR-HRV	HF-HRV
slope	-0.33±0.21	-0.20±0.03
intercept	1.92±0.31	1.87±0.11

4. Discussion

Results showed that a linear dependence between the fractal dimension and the power-law exponent of HRV series exists. However, the relationship was detectable only when the slope of the spectral density was calculated over the whole spectrum instead of considering more widespread very low frequency ranges. This implies that HRV is not characterized by a perfect $1/f^{\beta}$ spectrum, i.e. the underlying temporal process is not perfectly fractal in nature. The issue may lead to the ongoing debate around the possible multifractal nature of HRV series [13].

Both experimental relationships of normal and heart failure subjects didn't match the theoretical one valid for fBm processes (1). Moreover, the differences between the normal and the heart failure populations were significant. The reason behind the multiplicity of lines could lie, as suggested by Higuchi [10], in the power spectrum phase distributions of the two groups of HRV series. According to his findings, the phase distribution strongly affects the irregularity represented in terms of fractal dimension and therefore leads to a deviation from the fBm relationship of equation (1).

5. Conclusion

The fractal-like behaviour of heart rate variability can be equivalently characterized by either fractal dimension or power-law exponent. The fractional Brownian motion, assumed as a mathematical model for HRV, provides a linear relationship between β and FD (1) which is exploited by researchers to easily obtain one index from the other.

Preliminary results presented in this paper suggest that the theoretical relationship, valid for fBm, if empirically applied on HRV series, may lead to misleading results, in particular when the β index is calculated in the widespread very low frequency ranges. Even if the outcomes discourage the search for a single β/FD scaling relationship for the HRV of different categories of subjects, further investigations are required to better understand the underlying mechanisms of heart rate fluctuations. Before to conclude that fBm is not the best model for HRV, it could be interesting to extend the study to other populations (hypertension, post-myocardial infarction, heart transplanted, etc.).

However, for the time being, direct estimation of β and FD indices provides more correct values than those indirectly obtained by applying the theoretical relationship (1). The common practice of estimating FD from β for HRV applying the fBm relationship should be reconsidered.

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