

Heartbeat Dynamics from a Microcanonical Multifractal Approach

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Abstract

Heartbeat dynamics is a complex system. To characterize it properly, advanced nonlinear signal-processing methods are needed. In this context, recent developments on reconstructible signals and multiscale information content have led to the Microcanonical Multifractal Formalism (MMF). The MMF provides signal-analysis techniques particularly suited to heartbeat dynamics. In particular, electrocardiogram signals and electric potential in the endocardium allows detecting slow-changing transitions. Detecting regime transition could be used for early warning and treatment of cardiac arrhythmias. In this context, we present an application to the case of Atrial Fibrillation in which we detect distinctive parameters for the transition matrix.

1. Introduction

Heart rhythm is formed through complex synchronization processes between pacemaker cells and consequently displays chaotic rate fluctuations. These fluctuations are tiny compared to average interbeat intervals, so the sinus rhythm appear as mainly periodic, but fluctuations around this main period follow structured complex dynamics. Moreover, the characterization of these fluctuations is of vital importance in detecting signs of transition to an arrhythmia, despite appearing regular [1–4].

The human heart has a complex structure and a complex electrical activity. Cardiac action potential is led by polarization of pacemaker cells. These cells are not homogeneous, but mainly concentrate on nodes (sinoatrial and atrioventricular), and the Purkinje fibers that innervate the whole ventricular myocardium. The action of pacemaker cells controlling systoles and diastoles in an organized manner to ensure the optimal pumping [5].

Early studies of interbeat fluctuations found them to have a multifractal scale-invariant structure [6–8], which is the result of a complex synchronization process in the

hierarchical network of pacemaker cells [1]. The resulting signal reflects the network topology generating it, and that is why the MMF becomes especially suitable for the analysis of this dynamic structure. In particular, a analysis based on the singularity exponents and the optimal wavelet allows a direct access to the geometric characteristics of the multiscale behavior. This methodology is known to give more accurate estimation of the tails of the singularity spectrum and is generally more robust on empirical data. Having accurate estimates is of paramount importance to anticipate as much as possible when the heartbeat starts to drift from the healthy behavior.

Atrial fibrillation (AF) is the most common form of cardiac arrhythmia. It follows from the chaotic operation the upper heart (atria) and it can induce life-threatening complications such as heart failure or stroke. In some cases of AF, medication is ineffective and the treatment consists in radiofrequency ablation of the endocardial tissue to ease the cyclic electrical circulation. In case of paroxysmal AF, Haïssaguerre et al. have shown [9] that for 80 % of patients, electrical insulation of the pulmonary veins allows the patient to regain a normal heart rhythm [10–16], but in persistent or permanent AF, the location of pathogen areas remains difficult and is still an unsolved problem. In this context, non-linear analysis techniques such as MMF can be applied to identify dynamical changes that lead to recovery.

The paper is structured as follows: the next Section introduces the basics of the Microcanonical Multiscale Formalism (MMF) and the methodologies derived from it for proper signal processing. In Section 3 we present the analysis on atrial fibrillation data and discuss how it identifies dynamical changes in cardiac rhythm. We also present a reconstruction formula that naturally defines a way to sift a Markovian fast dynamics from a slow dynamics modulated by regime changes. Finally, in Section 4 we draw the conclusions of our work.

2. Microcanonical multiscale analysis

The Microcanonical Multiscale Formalism (MMF) is a theoretical and methodological framework for the analysis of multiscale signals. Its basic element of description are the singularity exponents, which are the exponents describing the local regular/singular behavior of the signal around each point. These are defined from the gradient-modulus measure of the signal [17].

A detailed definition and description of the MMF can be found in [17]. The basic definition is that given a signal $s(\mathbf{x})$, it has a singularity exponent $h(\mathbf{x})$ when the following equation holds:

$$\mathcal{T}_\Psi \mu(\mathbf{x}, r) = \alpha_\Psi(\mathbf{x}) r^{h(\mathbf{x}) + o(r^{h(\mathbf{x})})} \quad (r \rightarrow 0) \quad (1)$$

where $\mathcal{T}_\Psi \mu(\mathbf{x}, r) = \int_{\mathbb{R}} d\mu(\mathbf{x}') \Psi((\mathbf{x} - \mathbf{x}')/r)$ is the wavelet projection of the measure μ at point \mathbf{x} and scale r , $d\mu(\mathbf{x}) = \|\nabla s\|(\mathbf{x}) dx$ is the *gradient-modulus measure* and Ψ is a certain kernel known as *mother wavelet*.

3. Heartbeat analysis

The present work extends our preliminary findings in [18]. We have analyzed measures of the cardiac potential for six different patients in two heartbeat regimes: normal (sinus) rhythm and atrial fibrillation. Data consists of measures obtained through three catheter electrodes inside the heart during the radiofrequency ablation procedure, as well as electrocardiogram measures on the skin. Each dataset contains 21 channels (17 come from the catheters and 4 from the electrocardiogram), all of them consisting in electric potential differences sampled at 1 kHz, for a total of 1 183 232 data points.

The first step consists in verifying that eq. (1) holds for our signals, which means that our framework is appropriate for calculating the singularity exponents. Then, we define a dynamical model for heartbeat analysis by means of these singularity exponents and, in particular, their associated Most Singular Component (MSC), i.e., the set of points containing the most singular exponent. A key aspect here is that most of the dynamical information of the signal is contained only in its orientation over the MSC [19], so that we only need to characterize well this orientation.

Firstly, we determine where the MSC is located in the signal, that is to say, we locate where are the smallest (most singular) exponents. From this, we define the oriented MSC as a function that is zero everywhere except for the most singular points, where it takes the value +1 or -1 according to the sign of the gradient. If then we apply the reconstruction formula in [20] to our oriented MSC, the result is a *reduced* signal whose evolution coincides with the original signal at short scale. At longer scales,

there appears a slow divergence between original and reduced signal and this fact permits to model the heartbeat as a combination of a fast dynamics driven by the MSC orientation and a slow-varying field that modulates it [21].

The oriented MSC $\delta_\infty(t)$ takes 3 values (+1 on MSC points of positive derivative, -1 on MSC points of negative derivative and 0 on non-MSC points) and transitions from one point and the next are described as a Markov chain. Let σ be the Markov states, the two-point joint probability is expressed as:

$$P(\sigma_0, \sigma_\tau) = \langle P(\delta_\infty(t) = \sigma_0, \delta_\infty(t + \tau) = \sigma_\tau) \rangle \quad (2)$$

and the marginal probabilities:

$$P(\sigma_0) = \langle P(\delta_\infty(t) = \sigma_0) \rangle_t = \langle P(\sigma_0, \sigma_\tau) \rangle_{\sigma_\tau} \quad (3)$$

Because $P(\sigma_\tau, \sigma_0) = P(\sigma_\tau | \sigma_0) P(\sigma_0)$, if the process is Markovian then the two-step transition can be expressed as twice the one-step transition:

$$P(\sigma_2 | \sigma_0) = \sum_{\sigma_1} P(\sigma_2 | \sigma_1) P(\sigma_1 | \sigma_0) \quad (4)$$

i.e., all the dynamic steps are described only through one transition $P(\sigma_1 | \sigma_0)$. We compare the empirical $P(\sigma_2 | \sigma_0)$ to the one calculated with the preceding equation and find that they coincide. This verifies that the process is well described as a Markovian chain.

The one-step transition is matricially expressed as the transition matrix of the process, in our case:

$$T = \begin{pmatrix} 00 & 0+ & 0- \\ +0 & ++ & +- \\ -0 & -+ & -- \end{pmatrix} \quad (5)$$

One important point is that T^∞ results in the stationary distribution. This means that when T is applied on a stationary state the result does not change, so the first eigenvalue of the matrix is 1. The other two secondary eigenvalues are characteristic of the dynamics.

We have calculated the Markov processes for our data classified in four categories: internal signals under Atrial Fibrillation (AF), internal signals under sinus rhythm, ECG signals under AF and ECG signals under sinus rhythm. No significant differences have been detected from one channel to the other inside a category, meaning that it is appropriate to group them. In Table 1 we show the results. We see that there is a particular signature of AF that is conserved when the signal inside the heart is propagated to the skin. This suggests that it could be possible to finely monitor the AF evolution and severity from external ECG measures, and transitions to and from fibrillation could be immediately detected.

$\begin{pmatrix} .83 & .09 & .08 \\ .56 & .36 & .08 \\ .54 & .09 & .37 \\ & .29, .27 & \end{pmatrix}$	$\begin{pmatrix} .81 & .09 & .10 \\ .59 & .34 & .07 \\ .60 & .09 & .31 \\ & .25, .21 & \end{pmatrix}$
$\begin{pmatrix} .84 & .09 & .07 \\ .71 & .22 & .07 \\ .69 & .07 & .24 \\ & .17, .13 & \end{pmatrix}$	$\begin{pmatrix} .82 & .08 & .10 \\ .74 & .21 & .05 \\ .73 & .04 & .23 \\ & .17, .09 & \end{pmatrix}$

Table 1. Empirical transition matrices of the oriented MSC and their respective second and third eigenvalues for: internal under AF (top left), external under AF (top right), internal normal (bottom left), external normal (bottom right). The dynamical parameters in internal measures are still externally observed with minimal differences. The estimated eigenvalues have an uncertainty around ± 0.04 through propagation of the sampling error.

3.1. Source field

Since the oriented MSC drives the fast dynamics, the sign clustering determines a big part of the dynamical structure. On a longer scale, the MSC alone does not precisely reconstruct the signal because its amplitude factor slowly evolves. This amplitude is the source field: given a signal s and the reduced r constructed from its oriented MSC, the source field ρ is defined such that $\nabla s(x) = \rho(x) \nabla r(x)$. Nevertheless, since the reduced signal is not well defined outside the MSC, we need to estimate the source field in terms of measures [21]:

$$\mu_s(\mathcal{A}) = \int_{\mathcal{A}} d\mu_r(x) \rho(x) \quad (6)$$

This allows expressing the source field as the Radon-Nikodym derivative of two measures, $\rho(x) = d\mu_s/d\mu_r$.

Several strategies exist to numerically evaluate Radon-Nikodym derivatives. Since we are interested in the analysis of slow transitions, we use an iterative method to fit eq. (6) in a piecewise constant fashion. This has the advantage that the source field concentrates on the description of the dynamical borders and lets the MSC lead the fast Markovian-stable dynamics in-between. We show the results in Figure 1, where we can see how the source field varies infrequently and it exhibits sharp transitions at the same time. In consonance with the MSC orientation, we observe that the dynamical character in the case of AF is significantly different from under sinus rhythm.

We observe a correspondence of the transition points with the points in which the deviation between the original and the reconstructed series is more important. So these transition points correspond to transitions in the reconstructibility and in the content of information, which means that the detected transitions correspond to actual changes in the dynamical properties of the signal. The concrete mechanism that establishes the link of correspondence with the electrophysiological transitions is nontrivial and complex.

Figure 1. Source field and reconstruction for the V1 electrode. Top panels: source field displayed (solid) over the original signal (dashed) for the case of sinus rhythm (left panel) and atrial fibrillation (right panel). Bottom panels: signal reconstruction (dashed) based on the source field and the Markov-chain modeled MSC. Signs of Atrial Fibrillation are noticed in the dynamical parameters. High quality reconstruction, especially in peaks, means that we properly characterize the signal.

4. Discussion and conclusions

In this work, we have presented the study of cardiac fluctuations between beats under the approach of the Microcanonical Multiscale Formalism (MMF). To that extent, we geometrically characterize the resulting multiscale structure defined around singularity points. This way, we are able to decompose the signal into different regimes according to their characteristic dynamics. Singularity exponents directly characterize the information content of the component. Consequently, our analysis provides direct access to the dynamical structure at each point of the signal. When further exploited, this analysis shows that the most singular component (MSC) contains the information of the entire signal and can restore it. In other words, this component drives the dynamics of the signal.

In that sense, we found that the characteristic dynamical parameters retrieved from MMF analysis –namely the orientation of the MSC– can be dynamically described as a Markov chain. Furthermore, these dynamic parameters (in particular, the transition eigenvalues) under atrial fibrillation (AF) are significantly different from the sinus-rhythm case. Additionally, they can be equally detected from intracardiac electrodes or from standard electrocardiogram measures on the skin. Therefore, a possible application would be early detection of transitions to or from the AF.

Finally, the fact that the signal is reconstructible allows the determining of a slowly-varying source field that modulates the fast MSC Markov process. These source fields accurately describe the multifractal dynamic changes, what would suggest a possible relationship with transitions in electrophysiological processes, such as the evolution of the cardiac regulatory mechanism and changes in conductivity of the tissue.

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