Low Complexity Spectral Analysis of Heart-Rate-Variability through a Wavelet based FFT

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Abstract

In this paper, a low complexity system for spectral analysis of heart rate variability (HRV) is presented. The main idea of the proposed approach is the implementation of the Fast-Lomb periodogram that is a ubiquitous tool in spectral analysis, using a wavelet based Fast Fourier transform. Interestingly we show that the proposed approach enables the classification of processed data into more and less significant based on their contribution to output quality. Based on such a classification a percentage of less-significant data is being pruned leading to a significant reduction of algorithmic complexity with minimal quality degradation. Indeed, our results indicate that the proposed system can achieve up-to 45% reduction in number of computations with only 4.9% average error in the output quality compared to a conventional FFT based HRV system.

1. Introduction

Power spectrum density (PSD) is a positive real function of a frequency variable associated with a stationary stochastic process describing how the power of a signal or time series is distributed with frequency [1]. The analysis of PSD known as Power Spectral Analysis (PSA) provides a non-invasive means of accessing the physiological and pathological status of a person and is becoming a ubiquitous tool in the diagnosis, prediction and monitoring of various cardiac (i.e. sinus arrhythmia) and brain (i.e. epilepsy) malfunctions [2]. It is indicative that the ratio between the low frequency power (LFP) and the high frequency power (LFP) of cardiac spectra is used as quantitative indicator of many cardiac diseases (i.e. episodic hypertension, tachycardia, bradyarrhythmia, Guillain-Barre syndrome, etc.). Therefore, PSA, the main instrument for estimating such ratios, can play a significant role in diagnosing any malfunction and helping in providing suitable treatments based on the overall health status of a person.

Traditionally the PSA of rate variability involves the extraction of time intervals between successive heart beats (RR intervals) from a continuous ECG of a person and the estimation of a periodogram by using fast Fourier transform (FFT) or autoregressive (AR) modeling [3]. In these processes, as the RR-intervals are not evenly spaced data, the calculation of PSD requires re-sampling that is usually done by various interpolation methods. However, such methods alter the frequency contents of the signal due to the nonlinear low pass filtering effect [3].

To circumvent the issues of such methods the Lomb periodogram was introduced as a technique for deriving the PSD of unevenly spaced data without the need for interpolation and re-sampling [3]. While such a method provides theoretically a suitable procedure for estimating the PSA of unevenly spaced data, unfortunately, as in the case of FFT and AR methods, it requires a large number of computations hindering the use of such methods in portable health monitoring devices in which low power consumption is a very important requirement. Fortunately, a fast algorithm for evaluating Lomb and reducing the order of operations was proposed in [3]. However, Fast-Lomb may have reduced the number of computations of original Lomb by approximating the trigonometric sums using two complex FFTs, but unfortunately its complexity still remains high. Therefore, there is still a need for breakthroughs in order to reduce the complexity and thus the power consumption for allowing the realization of PSA on portable devices [4].

We develop a low complexity PSA system for HRV by reducing significantly the computations required in the Fast-Lomb algorithm. Our approach relies on the fact that cardiac signals have unique characteristics that were never being utilized for complexity reduction. Specifically, we observed that the heartbeats (RR-intervals) extracted by ECGs are sparse in the wavelet domain. While such an attribute was already taken into account by existing systems for compressing the raw cardiac signals and wirelessly transmitting them to external devices for processing them further [2, 7], it was not utilized for complexity reduction of PSA methods. The main reason is that the kernel of Fast-Lomb periodogram, the FFT cannot expose and utilize easily the sparsity hidden in RR intervals. We propose to implement the core-kernel of Fast Lomb, the FFT, using wavelets that can expose the sparsity of the signals and concentrate the signal energy into few factors termed as significant. Based on such a property we can prune several less-significant data that carry less signal energy and thus reduce the complexity of PSA. Overall, the contributions of the paper can be summarized as follows:

1) Analyze the disadvantages of traditional Fast-Lomb method associated with the inherent inability of its kernel (FFT) to utilize the properties of bio-signals.
2) Develop a Fast-Lomb method based on Discrete Wavelet Transform (DWT) for spectral analysis of HRV. The complexity of the new algorithm and the optimal parameters that allow significant reduction of the computations are being analyzed.
3) Show the trade-offs achieved between complexity and detection capability of sinus arrhythmia based on numerous cardiac samples.

The rest of the paper is organized as follows. Section 2 describes the basics of conventional PSA methods and analyzes the disadvantages of Fast-Lomb and its kernel the FFT. Section 3 presents the proposed approach while Section 4 describes the achievable trade-offs. Finally, conclusions are drawn in Section 5.

2. Conventional PSA methods of HRV

Traditionally the PSA of Heart Rate Variability involves the extraction of time intervals between successive heart beats (RR intervals) from a continuous ECG of a person and the estimation of a periodogram by a method called Lomb. In general, the Lomb periodogram computes the power of a given sinusoid signal $\mathbf{P_N(\omega_j)}$ as:

$$P_N(\omega) = \frac{1}{2\sigma^2}\left\{\frac{\left[\sum(x_j-\mu)cos\omega(t_j-\tau)\right]^2}{\sum cos^2\omega(t_j-\tau)^2} + \frac{\left[\sum(x_j-\mu)sin\omega(t_j-\tau)\right]^2}{\sum sin^2\omega(t_j-\tau)^2}\right\} \quad (1)$$

where $\boldsymbol{\mu}$ and $\boldsymbol{\sigma}$ are the mean and variance, respectively and $\boldsymbol{\tau}$ is a constant offset for each angular frequency $\boldsymbol{\omega_j}$ that makes the periodogram invariant to time-shifts.

Due the high complexity of the Lomb periodogram, a fast algorithm for evaluating it and reducing the order of operations was proposed in [3]. The Fast Lomb algorithm as it is called uses two complex FFTs in order to reduce the trigonometric sums (Eq. 1) to four simpler sums. However, such a method still requires large number of computations and comes with the main disadvantages of its core kernel (FFT module) that are described next.

Specifically, the kernel of Fast-Lomb, the FFT cannot take advantage of the sparse nature of the RR intervals, wasting in many cases computations which could otherwise be approximated with suitable changes. This main shortfall can be attributed to several issues:

i) The RR-intervals are sparse in the wavelet domain and as such FFT cannot utilize such sparsity.
ii) In the traditional FFT, all the twiddle factors in the butterfly operations are unit magnitude complex numbers [5]. Hence, all parts of the FFT structure are of equal importance and thus cannot concentrate the signal energy to few coefficients (as in case of DWT).
iii) Classical pruning methods may help in reducing the complexity of power-hungry FFT, but they do not work well when any non-zero inputs are randomly located as in some cases of extracted RR-intervals. In other words any sparse signal does not give rise to a faster FFT algorithm [2, 5].

To circumvent these issues and enable accuracy vs. complexity tradeoffs, we propose to implement the core-kernel of Fast Lomb, the FFT, using DWT that can expose the sparsity of RR intervals in the wavelet domain. DWT has twiddle factors of varying magnitude (not lying on unit circle as in case of FFT) that have the ability to concentrate the signal energy to few factors. Such a property can actually help us in classifying the data into significant/less-significant and ensure minimum quality degradation by dropping only insignificant data.

3. Proposed approach

In this section we describe the methods for the development of a low complexity spectral analysis tool.

3.1. Wavelet based FFT

To begin with, we observed that the kernel and the most computational part of the Fast Lomb algorithm is an *N* order FFT which can be written in a matrix form as:

$$\mathbf{F_N} = \mathbf{F_N}\, \boldsymbol{S'_N}\, \mathbf{S_N} = \begin{bmatrix} \boldsymbol{I_{N/2}} & \boldsymbol{T_{N/2}} \\ \boldsymbol{I_{N/2}} & -\boldsymbol{T_{N/2}} \end{bmatrix} \begin{bmatrix} \boldsymbol{F_{N/2}} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{F_{N/2}} \end{bmatrix} \mathbf{S_N} \quad (2)$$

where $\boldsymbol{T_{N/2}}$ is the diagonal matrix with twiddle factors on the diagonal and $\boldsymbol{S_N}$ is an NxN even-odd separation matrix. Interestingly, it was shown mathematically that an approximation of FFT using DWT is possible [5]. Specifically, similar to FFT matrix factorization (Eq. 2) the following factorization can be applied:

$$\mathbf{F_N} = \mathbf{F_N}\, \boldsymbol{W'_N}\, \mathbf{W_N} \begin{bmatrix} \boldsymbol{A_{N/2}} & \boldsymbol{B_{N/2}} \\ \boldsymbol{C_{N/2}} & \boldsymbol{D_{N/2}} \end{bmatrix} \begin{bmatrix} \boldsymbol{F_{N/2}} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{F_{N/2}} \end{bmatrix} \mathbf{W_N} \quad (3)$$

where $\boldsymbol{A_{N/2}}, \boldsymbol{B_{N/2}}, \boldsymbol{C_{N/2}}, \boldsymbol{D_{N/2}}$ are all diagonal matrices. The values on the diagonal of $\boldsymbol{A_{N/2}}$ and $\boldsymbol{C_{N/2}}$ are the length-N DFT of the lowpass filter of the wavelet transform whereas the values on the diagonal of $\boldsymbol{B_{N/2}}$ and $\boldsymbol{D_{N/2}}$ are the length-N DFT of the highpass filter of wavelet transform. Note also that $\mathbf{W_N}$ is the matrix form for a specific N-order wavelet basis.

The factorization shown in Eq. 3 suggests a DWT based FFT algorithm, whose block diagram for an order of N = 8 is depicted in Fig. 1. As it can be seen, the algorithm consists of 2 main stages; the highpass and the lowpass DWT outputs go through separate length-4 DFT, and then they are combined with butterfly operations. The same scheme shown in Fig. 1 is iteratively applied to shorter length DFTs (i.e., 4-point) to get the full DWT based algorithm. The full system is equivalent to a binary tree wavelet packet followed by modified FFT butterfly operations, where the twiddle factors are the frequency response of the wavelet (lowpass and highpass) filters.

3.2. Reducing the complexity

Even if the above algorithm allows the representation of FFT by using wavelets, which could help us overcome the shortcomings of the traditional PSA methods discussed in Section 2, it comes with a higher complexity than an original N-order FFT. To this end, we propose to reduce the complexity of such an algorithm which can serve as the core kernel of Fast Lomb method for the estimation of the trigonometric sums in Eq. 1. This is achieved by utilizing the sparse nature of the RR intervals in the wavelet domain [2] and applying approximations in the various stages of the algorithm, as discussed below:

a) We observed that the first stage of the algorithm consists of DWT, which has excellent time and frequency localization. Actually, each DWT stage by applying a high pass and a low pass filter computes the so-called approximation and detail coefficients, respectively that contain a percentage of the initial signal energy. As the stages (known as scales) increase the frequency (low frequencies) and time (high frequencies) resolution increases. By utilizing such a property we find that the first stage of the FFT-DWT algorithm (the DWT-decomposition stage) separates the processed RR intervals into significant (high magnitude values) and less-significant (low magnitude values) which can ultimately be dropped as highlighted in Fig-1. So in the first step of the algorithm, the complexity reduction is achieved by eliminating the high pass-detail computations, highlighted in red in Fig. 1. As it can be seen in Fig. 1, by dropping the high-pass band, the multiplications and additions in the second stage of the FFT can also be pruned.

b) In the second stage the DWT outputs are multiplied with twiddle factors that are the frequency response of the filter coefficients of the chosen wavelet basis (Haar, Db2, Db4 etc.). Such factors carry the unique property that they do not lie on the unit circle as opposed to the FFT twiddle factors,

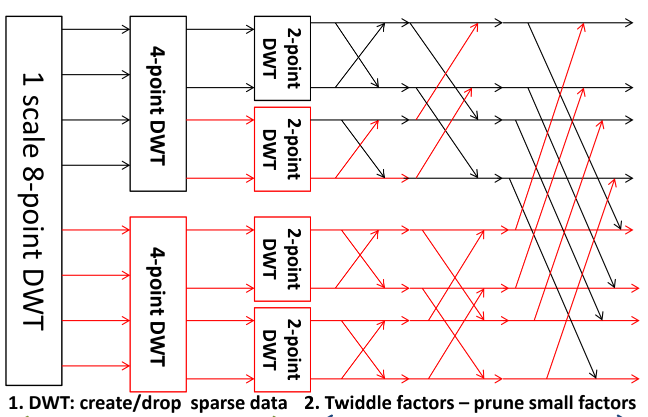

Fig.1. DWT-based FFT consists of two stages i) DWT, ii) Twiddle-factors. Insignificant data approximated/pruned in red.

meaning by that such factors differ in their magnitude significantly. Such a property allows us to distinguish the computations into significant/less-significant based on their values and contribution to the output-quality/end-result. By doing so we can prune further (less-significant) computations in the second stage of the DWT-FFT algorithm as highlighted in Fig. 1.

All in all, the new algorithm with the above approximations can be represented in a matrix form as:

$$\mathbf{F}_{N\_Pr} = \mathbf{F}_N \, W'_N \, \mathbf{W}_N = \begin{bmatrix} A_{N/2} & 0 \\ C_{N/2} & 0 \end{bmatrix} \begin{bmatrix} F_{N/2} & 0 \\ 0 & 0 \end{bmatrix} \mathbf{W}_N \quad (4)$$

where $B_{N/2}$ and $D_{N/2}$ (of Eq. 3) as well as the lower $F_{N/2}$ are set to zero due to their less-significant content. Further approximations are made as we discussed in the second stage based on the less-significant factors A, C. Note that the twiddle factors $[A_1, A_2 \ldots A_{N/2}]$ decrease in magnitude $(A_1 > A_2 \ldots . > A_{N/2})$, whereas factors $[C_1, C_2 \ldots C_{N/2}]$ increase in magnitude $C_1 < C_2 < \ldots < C_{N/2}$. Ideally $A_{N/2}$ and C_1, C_2 etc. have a value close to zero, so these coefficients can be removed from computations because of their lower significance to the computed output. It is also worth mentioning that the number of factors close to zero depends on choice of wavelet basis. As we go to a basis with higher filter sizes, the number of zeros is also increasing in the twiddle factors. However, at the same time the number of computations in the DWT stage is also increasing, so there is a trade-off between the approximations applied in the second stage and the number of computations in the DWT stage. Of course in this tradeoff we need to consider the quality degradation obtained due to the number of approximations. In any case, our approach provides the desired speed/power and accuracy tradeoffs while ensuring that the quality degradation will be minimum, since we eliminate only the less significant operations in each stage.

4. Results

In order to determine the optimum trade-off between quality and complexity that we discussed above, we have evaluated the complexity of the algorithm (of order 512) based on various wavelet basis (Haar, Db2, Db4) and the approximations applied in the two stages. Fig. 2(a) depicts the complexity in terms of the number of additions and multiplications for the various DWT bases and approximations in the first stage. We note that the FFT based DWT approach results in 46%, 59%, 86% computation increase compared to split-radix FFT in case that Haar, Db2 and Db4 are used as basis (without approximations) for the implementation of the algorithm, respectively. However, after dropping the high-pass parts that are termed as less-significant computations due the their low signal energy then the number of computations reduce by 28%, 21% and 8% compared to FFT in case that Haar, Db2, Db4 are used as DWT basis, respectively.

As we discussed in Section 3, in the proposed

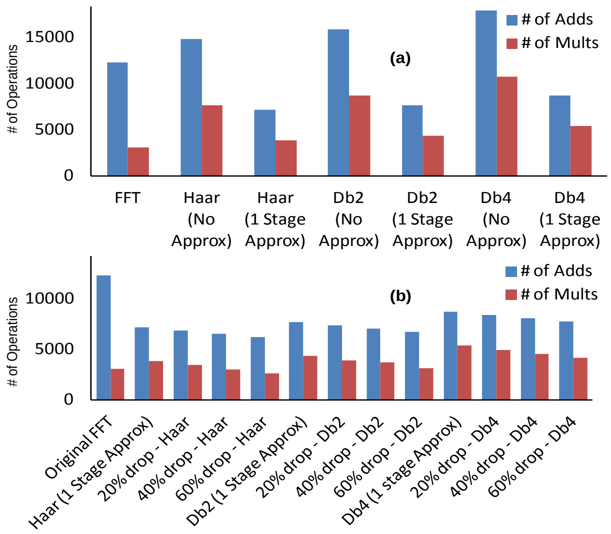

Fig.2. Complexity comparison of proposed approach with various Wavelet basis and approximations applied (a) in 1st stage and (b) in 2nd stage. (Reference 512-FFT Split Radix)

approach we apply further approximations in the 2nd stage of the algorithm. Fig. 2(b) compares the proposed approach to the conventional FFT approach in case of using various wavelet bases and for different degrees of approximations (Mode1-20%, Mode2-40%, and Mode3-60%) of less significant computations in the 2nd stage. It is evident that the choice of Haar wavelet basis clearly has the lowest complexity when compared to all other choices. Therefore, Haar was chosen as the wavelet basis for the implementation of the PSA system since it can lead to low-complexity. Overall, we observed that the proposed approximations can reduce the number of computations by up-to 45% (50% less additions and 15% less multiplications) compared to a conventional split-radix FFT algorithm.

In order to evaluate the output quality in case of the applied approximations we analyzed numerous sinus-arrhythmia samples from PhysioNet [6] and we evaluated the ratio between the low frequencies power (LFP) (0.04-015 Hz) and the high frequencies power (HFP) (0.15 – 0.4 Hz) of the heart rate spectra. In general, a dominant HFP or in other words a ratio LFP/HFP less than 1 indicate a sinus arrhythmia condition. Table 1 compares the ratio LFP/HFP for the different modes of approximations. Clearly the ratio remains close to the original value even when 45% of operations are pruned. The resulted PSA based on the proposed approach and the conventional Fast-Lomb method for a sample is depicted in Fig. 3. Interestingly, we can observe that even if we drop 45% of the computations we obtain only 3%

Table 1. LFP: HFP ratio comparison.

	Orig. FFT based HRV	HRV based on prop. FFT with 1st stage approx. and variousf approx. in 2nd stage			
		1st stage band drop	Mode1	Mode2	Mode3
Ratio	0.451	0.465	0.467	0.47	0.471

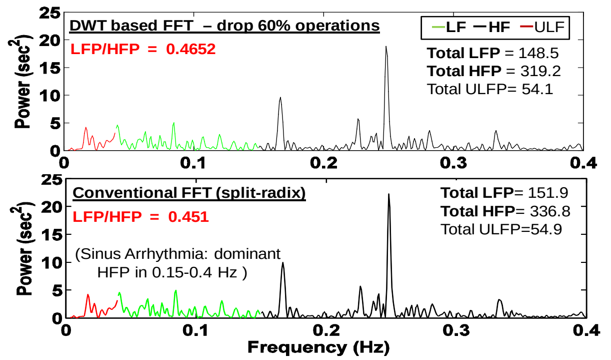

Fig. 3. Power Spectral Analysis – Conventional vs. Proposed

difference in LFP/HFP ratio compared to conventional PSA system, thus we can still easily identify the sinus arrhythmia condition. On average, we find that our approach can result in approximately 4.9% of error in LFP/HFP ratio after experimenting with numerous heart rate samples.

5. Conclusion

In this paper, a low complexity spectral analysis method was presented. It was shown that the proposed approach by utilizing the inherent properties of the cardiac signals and wavelets allows for significant complexity reduction with no sacrifice in output quality. All in all the proposed approach can reduce the power of the core kernel of PSA systems allowing the implementation of portable real time PSA of bio-signals.

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