

# Baseline Wander Removal in ECG and AHA Recommendations

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## Abstract

*Baseline wander is a kind of noise that affects all ECG signals. We have recently proposed a novel approach to its removal which is based on Quadratic Variation Reduction (QVR). The approach has very favorable properties and has shown to be effective in removing baseline wander, while preserving the ST segment level. It requires the determination of a detrending parameter. In this paper, we derive a linear time-invariant filter approximating QVR. The filter retains (approximately) the same optimality properties as QVR. Moreover, it provides a criterion for choosing the proper value of the parameter governing QVR, as a function of the spectral characteristics of baseline wander noise. Simulation results show that the filter is effective in removing baseline wander, while introducing minor distortion in the ST segment.*

## 1. Introduction

Baseline wander (BW) is a kind of noise affecting almost all bioelectrical signals and the electrocardiogram (ECG) is the worst affected in this regard. The main causes of BW in ECG signals are respiration, patient's movements, and fluctuations of the impedance between electrodes and skin [1]. BW is ubiquitous in all electrocardiographic devices and its removal is an unavoidable step in any processing of ECG signals [2].

BW in ECG is modeled as a low-frequency additive noise with band in the range  $0 \div 0.8$  Hz, extending up to 1 Hz, or even more, during stress tests [1]. As a consequence, BW and ECG have overlapping bands in the low-frequency region of the spectrum. Distortion in this band negatively affects the shape of the ST segment. This is a portion of the ECG that has strong clinical relevance related to the diagnosis of acute coronary syndrome, one of the most severe forms of heart disease and the main cause of mortality in developed countries [3]. The in-band nature of BW makes its removal difficult without affecting the ECG, when traditional techniques are used, thus spoiling relevant clinical information and leading to misdiagnosis of ACS [4].

The simplest approach to BW removal is high-pass fil-

tering with cut-off frequency of about 0.8 Hz [1, 5]. However, this approach, and in general any technique for BW removal that relies on frequency domain separation, may induce unacceptable artifactual distortions in the ST segment [4, 6]. To prevent this, the American Heart Association (AHA) recommends a cut-off frequency of 0.05 Hz for routine filters or 0.67 Hz or below for linear digital filters with zero phase distortion [6]. However, some residual BW may still be present in the filtered signal. Other common approaches to BW removal are adaptive filtering [7], median filtering [8], and wavelet adaptive filtering [9].

We have recently proposed a novel approach to BW removal that is based on Quadratic Variation Reduction (QVR) [10]. It estimates BW by means of a linear time-variant transformation that has some optimality properties. The approach has been shown to largely outperform state-of-the-art algorithms on BW removal [10–12], while preserving beat morphology, in particular the ST segment [11, 13]. Moreover, it is very fast [12]. In this paper, we derive a linear time-invariant (LTI) filter as an asymptotic approximation of the linear time-variant transformation performed by QVR. The LTI filter retains (approximately) the same optimality properties of QVR, and allows us to relate the value of the detrending parameter governing QVR to the spectral characteristics of the filter. This provides a criterion for choosing the proper value of such a parameter as a function of the spectral characteristics of BW noise. We carry out a comparative analysis of the filter with state-of-the-art approaches. Performance is assessed both in terms of effectiveness in removing BW and distortion of the ST segment.

The paper is organized as follows. The approach based on QVR is recalled in Section 2. The approximating filter is derived in Section 3. Sections 4 and 5 follow with simulation results and conclusions.

## 2. BW removal by quadratic variation reduction

BW noise is an additive low<sup>1</sup> “variability” component affecting the measured ECG. Thus, provided that we introduce a suitable index of “variability”, baseline can be es-

<sup>1</sup>Low with respect to ECG “variability”.

timated searching for the low variability component closest, in some sense, to the measured ECG. In [10, 12] we have proposed to quantify the variability of a generic vector through the *quadratic variation*.

Given a vector  $\mathbf{x} = [x_1 \cdots x_n]^T \in \mathbb{R}^n$ , the *quadratic variation* of  $\mathbf{x}$ , denoted by  $[\mathbf{x}]$ , is defined as

$$[\mathbf{x}] \doteq \sum_{k=1}^{n-1} (x_k - x_{k+1})^2 = \|\mathbf{D}\mathbf{x}\|^2, \quad (1)$$

where  $\|\cdot\|$  denotes the Euclidean norm and  $\mathbf{D}$  is the  $(n-1) \times n$  matrix with entries

$$\mathbf{D}_{ij} = \begin{cases} 1 & j = i \\ -1 & j = i + 1 \\ 0 & \text{otherwise} \end{cases}. \quad (2)$$

The quadratic variation is a well-known property used in the analysis of stochastic processes [14]. Moreover, it is a *consistent* measure of variability [10, 12]: for vectors affected by additive noise, on average it does not decrease and is an increasing function of noise variances.

Following the line of reasoning described at the beginning of this section, BW can be estimated searching for the component that is “close” to the observed signal, but has *reduced* quadratic variation. So, denoting by  $\mathbf{q}$  the vector of measured ECG, and by  $\mathbf{x}$  the vector of estimated baseline, in [10, 12] we propose to estimate the baseline  $\mathbf{x}$  by solving the following convex optimization problem

$$\begin{cases} \text{minimize} & \|\mathbf{x} - \mathbf{q}\|^2 \\ \text{subject to} & \|\mathbf{D}\mathbf{x}\|^2 \leq \rho \end{cases} \quad (3)$$

where  $\rho$  is a nonnegative constant that controls the quadratic variation of the estimated BW. It is possible to prove that the solution to problem (3) is given by

$$\mathbf{x} = (\mathbf{I} + \lambda \mathbf{D}^T \mathbf{D})^{-1} \mathbf{q} \quad (4)$$

where  $\mathbf{I}$  denotes the  $n \times n$  identity matrix, and  $\lambda$  is a nonnegative parameter that controls the quadratic variation of the solution  $\mathbf{x}$ , i.e., the degree of variability of the estimated baseline. For the details, see [10]. Finally, BW is removed by subtracting  $\mathbf{x}$  from the measured ECG.

It is worthwhile noting that the solution (4) is a linear operator acting on the signal to detrend  $\mathbf{q}$ . The matrix of the transformation, namely  $(\mathbf{I} + \lambda \mathbf{D}^T \mathbf{D})^{-1}$ , is not Toeplitz and this makes the transformation time-variant [15]. As a consequence, there exists *no* LTI filter that can achieve the same result as (4). This implies that it is not possible to associate a transfer function, or a frequency response, to the linear transformation (4). Thus, it is not easy to determine the proper value of the parameter  $\lambda$  as a function of the spectral characteristics of BW noise. To

cope with this problem, we will show in the next section that the linear time-variant transformation (4) can be well approximated by an LTI filter. The transfer function of the filter will provide us with a relationship between the spectral characteristics of the filter, e.g., bandwidth, and the parameter  $\lambda$ . This provides a criterion for choosing the proper value of this parameter as a function of the spectral characteristics of BW noise.

The approach based on QVR is interesting due to its good performance. Indeed, a thorough assessment of performance has shown that it outperforms competing algorithms for BW removal [10, 11], while preserving beat morphology, in particular the ST segment [11, 13]. It is also favorable in terms of computational complexity, which *linear* in the size of the vector to detrend.

In the next section we derive an LTI filter that approximates the transformation (4). The filter has the following major advantages: retains (approximately) the same optimality properties of QVR, and provides a criterion for choosing the proper value of the parameter  $\lambda$ .

### 3. The approximating LTI filter

The linear system (4) can be rewritten as

$$(\mathbf{I} + \lambda \mathbf{D}^T \mathbf{D}) \mathbf{x} = \mathbf{q} \quad (5)$$

where the matrix of coefficients  $(\mathbf{I} + \lambda \mathbf{D}^T \mathbf{D})$  is not Toeplitz. However, taking into account the structure of  $(\mathbf{I} + \lambda \mathbf{D}^T \mathbf{D})$  and writing explicitly the equations of the system (5), we get

$$(1 + \lambda)x_1 - \lambda x_2 = q_1 \quad (6)$$

$$-\lambda x_{i-1} + (1 + 2\lambda)x_i - \lambda x_{i+1} = q_i \quad (1 < i < n) \quad (7)$$

$$-\lambda x_{n-1} + (1 + \lambda)x_n = q_n \quad (8)$$

As  $n \rightarrow \infty$  the contribution of (6) and (8) tends to be less important and can be neglected. So, in this condition the system (5) can be approximated by (7) only, which is an LTI transformation.

Taking the Z-transform of (7), we get the following transfer function between  $q_i$  and  $x_i$

$$H(z) = -\frac{z^{-1}}{\lambda z^{-2} - (1 + 2\lambda)z^{-1} + \lambda} \quad (9)$$

which has the following reciprocal poles

$$p(\lambda) = \frac{2\lambda + 1 - \sqrt{4\lambda + 1}}{2\lambda} \quad \text{and} \quad p^{-1}(\lambda). \quad (10)$$

Since  $p(\lambda) \in [0, 1)$  for  $\lambda \in [0, +\infty)$ , the region of convergence (ROC) of  $H(z)$  to be stable is the ring  $\text{ROC} = \{z \in \mathbb{C} : p(\lambda) < |z| < p^{-1}(\lambda)\}$ . Thus (9) is a two-sided system with symmetric impulse response

$$h[n] = \frac{1-p}{1+p} p^{|n|}, \quad n \in \mathbb{Z}.$$

The transfer function (9) shows that BW removal by QVR (4) is approximated by a second order IIR filter with zero phase. The approximation holds true asymptotically for  $n \rightarrow \infty$ , but it is possible to prove that it is very good even for short ECG recordings ( $n$  of the order of  $10^2$ ).

The  $-3$  dB bandwidth of the filter in (9) is easily computed as

$$f_c = \frac{F_s}{2\pi} \arccos\left(1 - \frac{\sqrt{2} - 1}{2\lambda}\right) \quad (11)$$

where  $F_s$  is the sampling frequency. Formula (11) establishes a relationship between the filter bandwidth and QVR, providing us with a criterion for choosing the proper value of the parameter  $\lambda$ .

Moreover, it is possible to show that  $H(z)$  in (9) can be equivalently expressed as

$$H(z) = H_0(z)H_0(z^{-1}) \quad (12)$$

where

$$H_0(z) = \frac{1 - p}{1 - pz^{-1}}$$

is a stable single-pole IIR filter. As a consequence, using (12) can be equivalently seen as zero-phase filtering, where data are passed through a single-pole IIR filter in both the forward and reverse directions.

## 4. Simulation results

For a quantitative assessment of performance, we considered real BW free records and corrupted them with known baseline drift. ECG records from the MIT-BIH Arrhythmia Database [16] from PhysioNet [17] were considered. The database collects two-channel recordings acquired at a sampling frequency of 360 Hz with 11-bit resolution. In particular, we considered the first 5 minutes of the two channels of the record mitdb/119, which are mostly free from BW. The BW free records were manually segmented by an expert, who annotated the onset and offset of the ST segment of normal beats. A total of 247 ST segments was detected on the two BW free ECGs.

Synthetic BW was rendered as Gaussian white noise with variance  $\sigma^2 = 6.25$  low-pass filtered with bandwidth 0.8 Hz, following similar approaches in the literature [18]. We generated 30 independent realizations of synthetic BW, denoted by  $\mathbf{b}_i$ , with  $i = 1, \dots, 30$ . These were used to corrupt the 2 BW free ECG records, thus having 60 noisy ECG signals.

We compared the performance of the proposed filter with high-pass filtering (HPF) [1,6], adaptive filtering (AF) [7], and wavelet adaptive filtering (WAF) [9], which are common approaches to BW removal. The high-pass filter

is a linear-phase FIR filter synthesized applying the window method [15] using a Kaiser window, with 0.1 dB ripple in passband and 80 dB attenuation in stopband, with cut-off frequency 0.67 Hz compliant with AHA recommendations [6]. The convergence parameter of the adaptive filter and the wavelet adaptive filter is set to obey AHA requirements on cut-off frequency.

The quality of baseline wander removal is evaluated comparing the a priori known BW  $\mathbf{b}_i$  with its estimate through the following quantity

$$\varepsilon [\tilde{\mathbf{b}}_i(\text{alg}_k), \mathbf{b}_i] = \frac{\|\tilde{\mathbf{b}}_i(\text{alg}_k) - \mathbf{b}_i\|^2}{\|\mathbf{b}_i\|^2} \quad (13)$$

where  $\tilde{\mathbf{b}}_i(\text{alg}_k)$  denotes the baseline estimated by the  $k$ th algorithm, and  $\text{alg}_k \in \{\text{HPF}, \text{AF}, \text{WAF}, H(z) \text{ in (9)}\}$ .

Table 1 reports mean, standard deviation and median of baseline estimation error (13) on entire records ( $\mu_\varepsilon, \sigma_\varepsilon, \tilde{\varepsilon}$  computed over 60 noisy ECGs) and on ST segments of normal beats ( $\mu_\varepsilon^{\text{ST}}, \sigma_\varepsilon^{\text{ST}}, \tilde{\varepsilon}^{\text{ST}}$  computed over 14820 ST segments of normal beats). The parameter  $\lambda$  of the proposed filter was coarsely set to  $10^4$  after visual inspection of the detrended records. This value of  $\lambda$  corresponds to a cut-off frequency of 0.37 Hz, compliant with AHA recommendations. As Table 1 shows, the proposed filter exhibits the best performance among the approaches considered, both in terms of effectiveness in removing BW and distortion introduced in the ST segment.

The proposed filter is robust to variations of the detrending parameter and performs well for a wide range of values of  $\lambda$ . In this regard, we computed the performance with  $\lambda$  such that the resulting cut-off frequency is *not* compliant with AHA recommendations. In Table 2 we report the mean, standard deviation, and median of baseline estimation error (13) computed with  $\lambda = 2500$ , which corresponds to a cut-off frequency of 0.74 Hz, i.e., *not* compliant with AHA recommendations. Comparing the results of Table 2 with the first three rows of Table 1, the following considerations are in order. The baseline estimation errors on entire records introduced by the proposed filter are lower, in terms of mean, standard deviation, and median, than the errors returned by AF and WAF. The high-pass filter returns errors on entire records lower than the proposed filter. However, it should be noted that we deliberately considered a quasi-ideal high-pass filter, thus having high order. In clinical practice the filters used are of much lower order, with a consequent worsening of performance in removing BW. Further, with respect to the distortion of the ST segment, the proposed filter, although violating AHA recommendations, introduces minor distortion in the ST segment. In particular, the high-pass filter, which has the best performance in terms of baseline estimation error on entire records, introduces the greatest distortion in

	$\mu_\epsilon$	$\sigma_\epsilon$	$\tilde{\epsilon}$	$\mu_\epsilon^{ST}$	$\sigma_\epsilon^{ST}$	$\tilde{\epsilon}^{ST}$
HPF	0.86	0.28	0.84	25.14	570.09	0.76
AF	1.31	0.48	1.24	16.20	267.48	0.82
WAF	1.07	0.42	1.02	11.77	218.46	0.55
$H(z)$	0.60	0.16	0.59	9.76	164.47	0.43

Table 1. Mean, standard deviation, and median of baseline estimation error (13) over: entire records ( $\mu_\epsilon$ ,  $\sigma_\epsilon$ ,  $\tilde{\epsilon}$ ), and ST segment of normal beats ( $\mu_\epsilon^{ST}$ ,  $\sigma_\epsilon^{ST}$ ,  $\tilde{\epsilon}^{ST}$ ).

	$\mu_\epsilon$	$\sigma_\epsilon$	$\tilde{\epsilon}$	$\mu_\epsilon^{ST}$	$\sigma_\epsilon^{ST}$	$\tilde{\epsilon}^{ST}$
$H(z)$	0.96	0.44	0.89	9.86	175.90	0.35

Table 2. Mean, standard deviation, and median of baseline estimation error (13) with  $H(z)$  having cut-off frequency not compliant with AHA recommendations.

ST segment among the algorithms considered, although it is compliant with AHA recommendations.

## 5. Conclusions

In this paper, we derived an LTI filter approximating the approach to BW removal based on QVR, which has very favorable properties. The filter is a second order IIR filter with zero phase, and is equivalent to a stable single pole IIR filter applied in both the forward and reverse directions. The proposed filter constitutes an (approximately) equivalent implementation of baseline wander removal by QVR, thus retaining (approximately) the same optimality properties. Moreover, it provides us with a criterion for choosing the proper value of the parameter governing QVR, as a function of the spectral characteristics of BW noise. Simulation results show that the filter is effective in removing baseline wander, while introducing minor distortion in the ST segment, even when it is *not* compliant with AHA Recommendations.

## References

- [1] Sörnmo L, Laguna P. Bioelectrical Signal Processing in Cardiac and Neurological Applications. Elsevier Academic Press, 2005.
- [2] Bailey JJ. The triangular wave test for electrocardiographic devices: A historical perspective. J Electrocardiol 2004; 37(Suppl.):71–73.
- [3] Califf RM, Roe MT. ACS Essentials. Jones and Bartlett Publishers, 2010.
- [4] Brewer AJ, Lane ES, Ross P, Hachwa B. Misdiagnosis of perioperative myocardial ischemia: The effects of electrocardiogram filtering. Anesth Analg 2006;103(6):1632–1634.
- [5] Van Alsté JA, Schilder TS. Removal of base-line wander and power-line interference from the ECG by an efficient FIR filter with a reduced number of taps. IEEE Trans Biomed Eng 1985;32(12):1052–1060.
- [6] Kligfield P, Gettes LS, Bailey JJ, Childers R, Deal BJ, Hancock EW, van Herpen G, Kors JA, Macfarlane P, Mirvis DM, Pahlm O, Rautaharju P, Wagner GS. Recommendations for the standardization and interpretation of the electrocardiogram: Part I: The electrocardiogram and its technology. Circulation 2007;115:1306–1324.
- [7] Thakor NV, Zhu YS. Applications of adaptive filtering to ECG analysis: Noise cancellation and arrhythmia detection. IEEE Trans Biomed Eng 1991;38(8):785–794.
- [8] Hiasat AA, Al-Ibrahim MM, Gharaibeh KM. Design and implementation of a new efficient median filtering algorithm. IEE Proc Vis Image Signal Process 1999; 146(5):273–278.
- [9] Park KL, Lee KJ, Yoon HR. Application of a wavelet adaptive filter to minimise distortion of the ST-segment. Med Biol Eng Comp 1998;36(5):581–586.
- [10] Fasano A, Villani V, Vollero L. Baseline wander estimation and removal by quadratic variation reduction. Proc 33rd Int Conf IEEE Eng Med Biol Soc EMBC 2011;977–980.
- [11] Fasano A, Villani V. ECG baseline wander removal and impact on beats morphology: A comparative analysis. Computers in Cardiology 2013;.
- [12] Fasano A, Villani V. Baseline wander removal for bioelectrical signals by quadratic variation reduction. Submitted for publication 2013;.
- [13] Fasano A, Villani V, Vollero L. Fast ECG baseline wander removal preserving the ST segment. Proc 4th Int Symp Appl Sci Biomed Commun Tech ISABEL 2011;.
- [14] Shreve SE. Stochastic Calculus for Finance II: Continuous-Time Models. Springer Science+Business Media, Inc., 2004.
- [15] Oppenheim AV, Schaffer RW, Buck JR. Discrete-Time Signal Processing. 2nd edition. Prentice-Hall, Inc., 1999.
- [16] Moody GB, Mark RG. The impact of the MIT-BIH arrhythmia database. IEEE Trans Biomed Eng 2001; 20(3):45–50.
- [17] Goldberger AL, Amaral LAN, Glass L, Hausdorff JM, Ivanov PC, Mark RG, Mietus JE, Moody GB, Peng CK, Stanley HE. PhysioBank, PhysioToolkit, and PhysioNet: Components of a new research resource for complex physiologic signals. Circulation 2000;101(23):e215–e220.
- [18] Blanco-Velasco M, Weng B, Barner KE. ECG signal denoising and baseline wander correction based on the empirical mode decomposition. Comp Biol Med 2008;38:1–13.

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