

Preliminary Implementation of Novel Bifurcation Pressure Loss Model in a Reduced-Order Cardiovascular Flow Model

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Abstract

Reduced-order models are valuable tools in cardiovascular modeling when high-fidelity simulations are prohibitively expensive, but they often suffer reduced accuracy due to the simplifications in their formulations. One such assumption is that pressure is constant over a bifurcation. The resistor-resistor-inductor model was recently proposed to predict the pressure difference over bifurcations as a function of flow rate, using coefficients extracted from the bifurcation geometry using machine learning. We implemented the resistor-resistor-inductor model (previously only tested on isolated bifurcations) in a reduced-order cardiovascular model for an idealized test geometry and compare the results to those achieved with standard bifurcation handling. We found improved accuracy (defined by comparison to high-fidelity simulations) for both steady and transient flows.

1. Introduction

Computational models of cardiovascular flows have proven to be a valuable tool in the study and treatment of cardiovascular disease. They have been used for diagnosis, surgical planning, investigation of the mechanisms driving disease initiation and progression, and the engineering of medical devices [1]. Traditional high-fidelity cardiovascular models use finite-element techniques to solve the three-dimensional (3D) Navier Stokes equations over the vasculature. However, this computationally intensive task requires significant computational resources and long wait times, limiting practicality for some use cases.

Reduced-order models (ROMs) solve simplified versions of the Navier-Stokes equations to characterize cardiovascular flows at extremely low computational cost and are therefore useful for applications requiring many simulations (e.g. setting initial and boundary conditions, parameter estimation, optimization, and uncertainty quantification) or quick turnaround times (e.g. control systems for experimental setups operating in real-time and real-time surgical planning for diseases like Tetralogy of Fallot or

aortic coarctation)[1].

Although they are invaluable and widely used tools, ROMs are limited in accuracy and, therefore, in utility by their simplified formulations. One source of inaccuracy is the handling of pressure differences over vascular bifurcations [2]. These differences can be significantly affected by complex 3D flow behavior not captured in the lower-dimensional representations generally used by ROMs.

In this work, we analyze the resistor-resistor-inductor (RRI) model, which was recently proposed as a method to account for pressure difference over bifurcations in reduced order models [3]. This model was validated on isolated bifurcations, but has not previously been used in a reduced order model. We introduce a zero-dimensional (0D) reduced order model and discuss the standard handling of bifurcations. We then discuss the RRI model and its integration into the 0D model. We test the 0D model with standard bifurcation handling (hereafter referred to as “0D standard”) and the 0D model augmented with the RRI model (hereafter referred to as “0D RRI”) on an idealized vascular geometry. We compare the accuracy of the two bifurcation handling methods, using a high-fidelity 3D simulation as ground truth. Finally, we consider the implications of the improved bifurcation handling enabled by the RRI model and identify pathways to increase the accuracy and applicability of the RRI model.

2. Methods

2.1. 0D Model

This work considers a 0D ROM that represents vascular flows as electric circuits. In this formulation, flow and pressure are analogous to current and voltage, respectively, and the vessels are represented using circuit elements such as resistors, capacitors, and inductors [4]. The solution of the system of equations describing the electric circuit analog of the vascular flow yields a reduced-order approximation of bulk flow and average pressure at the nodes of the circuit. Hereafter, we refer to these quantities as flow (Q) and pressure (P). We use `svZeroDSolver` implemented in the open-source cardiovascular simulation soft-

ware suite, `SimVascular`.¹

2.2. Bifurcation Handling

In the “standard” handling of bifurcations in the 0D model, static pressure is assumed to be continuous over the bifurcation, that is

$$\Delta P = P_{\text{outlet}} - P_{\text{inlet}} = 0. \quad (1)$$

This is the default bifurcation handling implemented in `svZeroDSolver`, and is commonly used in the wider cardiovascular modeling community [5].

In the recently proposed resistor-resistor-inductor (RRI) model, the pressure difference over a bifurcation is modeled as the voltage drop over a serially connected resistor, quadratic resistor, and inductor, as follows

$$\Delta P = R_{\text{lin}}(\mathcal{G})Q + R_{\text{quad}}(\mathcal{G})Q^2 + L(\mathcal{G})\dot{Q}. \quad (2)$$

The linear resistance, quadratic resistance, and inductance are determined from the bifurcation’s geometry using a neural network. The bifurcation geometry is described by a vector containing the inlet radius, outlet radius, auxiliary outlet radius, outlet angle, auxiliary outlet angle, outlet length, and auxiliary outlet length, $\mathcal{G} = [r_{\text{inlet}}, r_{\text{outlet},1}, r_{\text{outlet},2}, \theta_1, \theta_2, l_1, l_2]$. As described in [3], two bifurcation types are considered, one representing the brachiocephalic bifurcation in the aorta and one representing bifurcations in the pulmonary vasculature. A neural network for each type was trained on a cohort of synthetically generated geometries representative of the bifurcation type. Note that the neural networks are only trained on individual bifurcations and are completely agnostic to flow through more extensive geometries, such as the test geometry for which we report numerical results in Section 3.

Each cohort comprised approximately 100 bifurcations whose geometric parameters were randomly sampled from uniform distributions ranging from the 40th to 60th percentiles of the parameter values observed for brachiocephalic and pulmonary bifurcations in the Vascular Model Repository.² Given the geometric parameters, the bifurcations were constructed using `SimVascular`’s Python API, and flow through them was simulated using `SimVascular`’s `svSolver`. For each bifurcation, two steady-state simulations were used to determine the values of R_{lin} and R_{quad} , and a transient simulation was used to find L .

2.3. Test Geometry

We tested the performance of `svZeroDSolver` using both the standard continuous pressure assumption Equation (1) (0D standard) and the RRI model Equation (2)

(0D RRI) on a test geometry. The test geometry, shown in Figure 1, consisted of a brachiocephalic bifurcation with a pulmonary bifurcation appended to one of the outlet branches, again constructed using `SimVascular`. The RRI model resistances and inductances for each bifurcation were determined from the bifurcation geometry using the neural networks trained on the cohorts described above. To isolate the effect of the bifurcation handling, there is no additional vessel segment connecting the two bifurcations. A prescribed inflow is applied at the inlet, and resistances and distal pressures are applied at the outlets.

Notably, the definition of a bifurcation differs between the RRI bifurcation model and the standard bifurcation handling implemented in `SimVascular`. In the standard handling, bifurcations are defined based on the intersection of centerlines while in the RRI model, the inlet is defined to be ≈ 5 inlet diameters upstream of the bifurcation point, and the outlets are defined to be ≈ 10 inlet diameters downstream of the bifurcation point, where the flow was generally observed to have returned to a fully developed velocity profile. The respective divisions of the test geometry into vessels and bifurcations are shown in Figure 1. The RRI implementation contains no vessels, while the standard implementation includes vessels whose characteristic resistances and inductances were computed according to [2].

0D simulations with both standard and RRI bifurcation handling were run both at steady state and for the transient flow profile shown in Figure 3. Transient simulation results are from a single cycle (preliminary studies conducted on all the models considered showed negligible difference across cycles). The chosen flow rates are similar to those observed at systole in the ascending aorta, and represent a “worst case”, high-flow, scenario where bifurcation losses are most impactful. We also ran 3D simulations to act as ground truth against which to compare 0D results. The 3D mesh was generated with `SimVascular` using boundary layer mesh refinement, mesh refinement around the bifurcation point, and a nominal mesh edge size chosen by mesh convergence studies. The flows through the 3D geometries were simulated using `SimVascular`’s stabilized finite element solver, `svSolver`, to solve the 3D incompressible Navier-Stokes equations. The 3D simulations took about 140 minutes on 48 nodes of a high-performance computing cluster and the 0D simulations took less than a second on a personal laptop.

3. Results

We observe a significant improvement from using the RRI bifurcation model instead of the standard bifurcation handling at steady state. In Figure 2, we show the pressure and flow predictions of the two 0D model implementations compared to the 3D simulation results. The 0D RRI model

¹<https://github.com/SimVascular/svZeroDSolver>

²<http://www.vascularmodel.com> (2022)

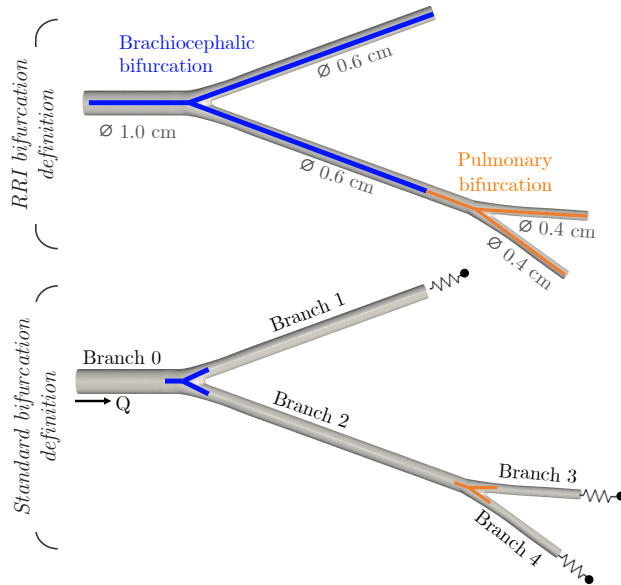


Figure 1. Idealized geometry on which the RRI 0D, standard 0D, and 3D simulations are run. The upper panel shows the RRI bifurcation definition, lists the vessel diameters, and identifies the brachiocephalic-based bifurcation and the pulmonary-based bifurcation. The lower panel shows the standard bifurcation definition, labels the branches of the geometry, and indicates the boundary conditions applied at the inlet and outlets.

tracks pressures and flows of the 3D simulation much better than the 0D standard model, which underestimates the pressure drops over the bifurcations. Notably, the 0D standard model underestimates the resistance of the branch containing the second bifurcation (branch 2), which causes it to over-predict the amount of flow going through branch 2. This, in turn, leads to an overestimation of the pressure at the outlets of branches 3 and 4 and an underestimation of the pressure at the outlet of branch 1. Note that the RRI formulation of the test geometry includes only bifurcations and no vessels - for this reason the 0D RRI results appear as points rather than lines in Figure 2.

We analyze the same geometry under transient flow. Since a flow boundary condition was prescribed at the inlet and pressure-related boundary conditions were prescribed at the outlets, we observed the largest pressure variability at the inlet. For this reason, we show the flow rate (same for all models) and pressure at the inlet for the two 0D implementations and the 3D solution in Figure 3. Although it does not track the 3D solution exactly, as in the steady case the 0D RRI model reproduces the 3D solution more closely than the 0D standard model. Note that, as expected, there is a phase-lag between the flow profile and pressure profiles due to inertial effects.

In Table 1 we list the pressure errors of the 0D RRI

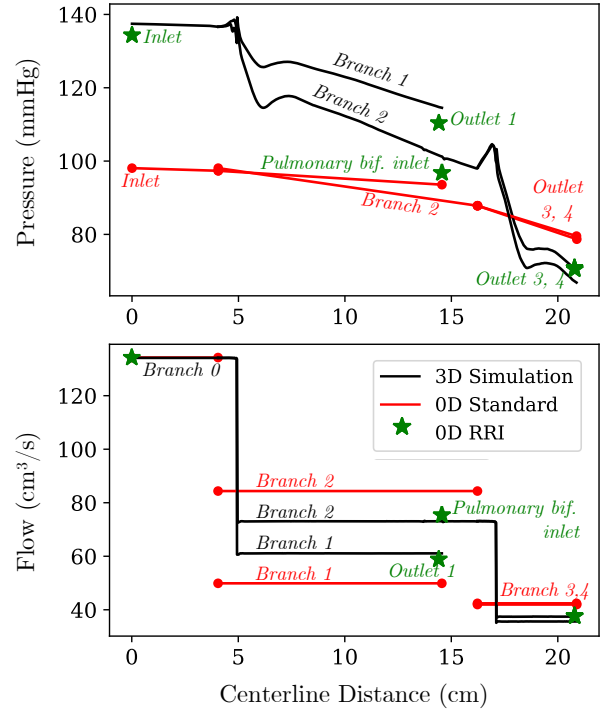


Figure 2. Steady state average pressure (top) and bulk flow (bottom), as predicted by the 0D RRI model, 0D standard model, and 3D simulation along the centerlines of the test geometry.

model and 0D standard model at the inlets and outlets of the test geometry. As expected, the errors were the most extreme at the inlet, where the most error had accumulated from failure to account for bifurcation pressure losses. The RRI 0D model consistently outperformed standard 0D model. Notably, the standard 0D model had a similar error for the transient and steady flows, while the 0D RRI model had a significantly larger error in the transient case than in the steady case.

4. Discussion

As expected, the inclusion of the RRI model in the 0D model increased its accuracy for a simple test case. This was true for both steady and transient flows, although in the transient case, the 0D RRI model did not perform as accurately as it did in the steady case. These results suggest the potential of the RRI model to significantly increase the accuracy of ROMs, particularly for geometries with high flow velocities and many bifurcations.

To further characterize the effects of including the RRI model in the 0D model, realistic test geometries (potentially drawn from patient-specific vasculatures) should be considered. However, several improvements to the RRI model are necessary before it can be used on “real-world”

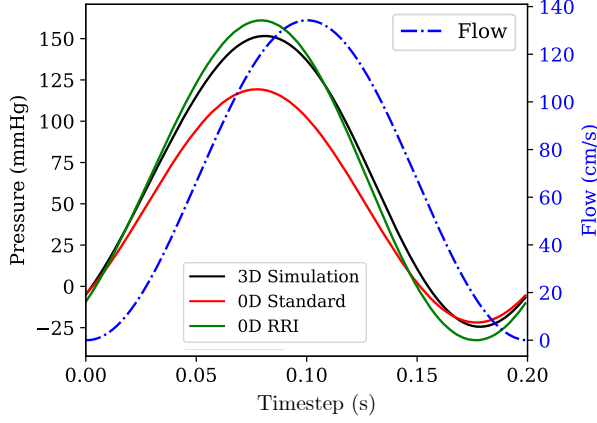


Figure 3. Pressure over time at the test geometry inlet predicted by the 0D RRI model, 0D standard model, and 3D simulation. The blue flow profile is prescribed at the inlet.

Total (mmHg) and Relative Pressure Error		
	RRI	Standard
Steady		
<i>Inlet</i>	3.1 (2.2%)	39 (29%)
<i>Branch 1 Outlet</i>	4.2 (3.6%)	20 (18%)
<i>Branch 3 Outlet</i>	0.27 (0.38%)	8.4 (12%)
<i>Branch 4 Outlet</i>	4.1 (6.1%)	13 (19%)
Transient		
<i>Inlet</i>	13 (7.2%)	39 (23%)
<i>Branch 1 Outlet</i>	16 (14%)	17 (15%)
<i>Branch 3 Outlet</i>	8.0 (12%)	7.8 (12%)
<i>Branch 4 Outlet</i>	8.3 (12%)	11 (17%)

Table 1. Errors of the 0D RRI model and 0D standard model at the inlets and outlets of the test geometry, defined as the absolute value of the difference between the 3D and 0D pressure value.

bifurcations.

First, the machine learning components of the RRI model must be trained on a much larger and more varied dataset covering the full range of bifurcations observed in the body. Training on bifurcations with more variable inlet and outlet lengths would also allow the RRI model to accommodate standard definition of bifurcations and vessels. Second, more sophisticated machine learning techniques could further improve RRI model. For instance, providing the entire 3D geometry to the machine learning model rather than the simpler bifurcation geometry vector $\mathcal{G}$ would enable the model to capture more geometric features to which it is currently blind (e.g., a stenosis at the center of the bifurcation). Such an approach would likely benefit from an alternative ML architecture, like a convolutional or graph neural network, as in [6]. A graph neural network structure would also enable the RRI model to han-

dle junctions with arbitrary numbers of inlets and outlets, commonly observed in the body.

5. Conclusions

We used the recently proposed RRI model for pressure differences over vascular bifurcations in a 0D cardiovascular flow model for an idealized test geometry. We found that, compared to the standard 0D model, the 0D model augmented with the RRI model produced results closer to those of a high-fidelity 3D simulation. While work remains to be done to make the RRI model widely applicable, this is a promising demonstration of the RRI model's capacity to boost the accuracy of reduced-order cardiovascular models.

Acknowledgments

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