Performance of Iterative Methods of ECGI in the Atria

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Abstract

The data completion problem of electrocardiographic imaging (ECGI), i.e., computing pericardial potentials from the body surface potentials, is a challenging task. In this context, many different methodologies can be used to solve this problem. These are divided into direct method (Tikhonov regularization) and indirect methods (iterative methods).

Employing realistic simulations, we compare three iterative methods: the Landweber method, the Algebraic Reconstruction Technique (ART), and the Range Restricted Generalized Minimal Residual method (RRGMRES), against the standard Zero Order Tikhonov regularization. Our analysis focuses on five atrial ectopics and five atrial reentries simulations, assessing the methodologies based on Correlation Coefficient (CC) of electrograms (EGMs) and Local Activation Times (LATs).

Iterative methods show promise in ECGI, obtaining equal or better results than Tikhonov regularization. However, due to the need of an independent stopping criteria Tikhonov regularization still leads on real applicability, suggesting its continued value in cardiac diagnostic applications despite the emerging potential of iterative approaches.

1. Introduction

Electrocardiographic imaging (ECGI) is a technology that allows obtaining estimations of the heart's electrical behavior without the need for invasive measurements. By modeling the propagation of the electric potential through the passive volume of the body [1], we can estimate pericardial activity by making use of the body surface potentials.

This ECGI formulation gives rise to an inverse problem in the form of a linear data completion Cauchy problem that relates the unknown pericardial potentials to the body surface potential recordings [2]. Several numerical methods exist to obtain a discrete relationship, usually referred to as the transfer matrix [3]. In this study, we employ a Boundary Element Method formulation [4]. Then, the unknown pericardial potentials are reconstructed trough a discrete inverse problem. Despite ECGI's ability to currently provide clinical diagnostics, it lacks precision for various heart disorders due to the ill-posedness of the underlying inverse problem.

The ill-posedness arises from the non-invasive measurements registered away from the region of interest, mathematically manifesting as a non-continuous dependence of the pericardial potential respect to the torso potentials. This leads to numerical errors, augmented by electrical noise or geometric inaccuracies, which can significantly distort the reconstruction [5].

To overcome this error and obtain a reliable solution, we must employ regularization methods. There are numerous regularization methods that can be employed in the linear discrete inverse problem of ECGI [6]. Various studies in recent literature explore some regularization methods [7–9] to solve the problem.

In this present study, three different iterative methods (the Landweber iteration, the Algebraic Reconstruction Technique (ART), and the Range Restricted Generalized Minimal Residual method (RRGMRES)) are compared with respect to the classical approach in ECGI, Zero Order Tikhonov regularization. The comparison is conducted in five atrial ectopics and five atrial reentries simulations by computing the Correlation Coefficient (CC) of the reconstructed Electrograms (EGMs) and Local Activation Times (LATs).

2. Methods

2.1. Dataset

Both BSPMs and EGMs were obtained using realistic computerized model of atria geometry from which a pericardial shell is created ($m = 4000$ nodes) and different models of torso geometries. For our initial tests, we con-

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duct simulations of five atrial ectopics and five atrial reentries. The resulting signals are contaminated with a signal-to-noise ratio (SNR) of 20dB of white Gaussian noise.

2.2. Data Completion Problem

We assume that the quasi-static approximation of Maxwell's equations holds for the electromagnetic fields considered. Then, the inverse problem can be formulated as a linear data completion Cauchy problem aiming to estimate the electric potential at the inaccessible pericardial heart's surface (or outer layer of the heart), i.e.,

$$\begin{aligned} -\nabla^2\phi(x) &= 0, && x \in \Omega \\ \partial_{n_T}\phi(x) &= 0, && x \in T \\ \phi(x) &= b(x), && x \in T \end{aligned}$$

where ϕ is the electric potential, b are the body surface potentials, and Ω is the domain encompassing the pericardium H and the torso T. Moreover, we are assuming a homogeneous medium.

There are different approaches that can be used to construct the linear operator used in the data completion problem. One of the most common approaches in clinical scenarios, the one employed in this study, is based on the Boundary Integral Equation formulation [3,4] (also named as Boundary Element Method (BEM)), where single and double layer operators are utilized to establish the relationship between torso and pericardial potentials. By discretizing the operator relationship between the pericardial and torso potentials, we are able to obtain a transfer matrix $A \in \mathbb{R}^{n\times m}$, which can be used in later steps. Note that we will only employ the rows related to electrode measures.

2.3. Discrete Regularization Methods

Once we have obtained the transfer matrix $A \in \mathbb{R}^{n\times m}$, we aim to estimate the electrograms $X \in \mathbb{R}^{m\times t}$ that generate our set of measures $B \in \mathbb{R}^{n\times t}$. However, due to the ill-posed nature of our problem, direct solvers cannot be employed. This can be seen in the exponential decay of the singular values of the transfer matrix.

The classical approach employed in ECGI consists of a Tikhonov regularization framework. This technique explicitly incorporates a regularization term

$$X^{\text{Tikh}} = \min_X \left(||AX-B||^2_{\text{Fro}} + \lambda||RX||^2_p\right)$$

where the regularization parameter, $\lambda \in \mathbb{R}$, aims to balance the residual error with its respective norm; R is a matrix that sets an a priori spatial regularization (by giving a weight to the regularization parameter for each component of X); and p corresponds to the l_p-norm (chosen depending on the a priori information known about the solution). In particular, Zero Order Tikhonov regularization [8] assumes an identity matrix for R (the regularization parameter acts equally on all components of X) and an l_2-norm. Moreover, since we are working with multiple time instants, we employ a Frobenius norm instead of the l_2-norm. Consequently, the Zero Order Tikhonov regularization corresponds to the formulation:

$$X^{\text{Tikh}} = \min_X \left(||AX-B||^2_{\text{Fro}} + \lambda||X||^2_{\text{Fro}}\right)$$

Additionally, in this study, we employ the L-curve method [10] on a predefined range of lambda values to choose the value of the regularization parameter.

Apart from classical Tikhonov regularization, we can employ iterative methods to solve the discrete inverse problem [6]. Among them, we evaluate in this study the Algebraic Reconstruction Technique (ART), the Landweber method, and the RRGMRES method. Notice that in all the iterative methods each time instant is reconstructed independently.

- The *Algebraic Reconstruction Technique (ART)* is a row-action method, i.e., each iteration consists of a sweep through the rows of the matrix $A \in \mathbb{R}^{n\times m}$.

- The *Landweber method* is an iterative method of the form

$$X^{(k+1)} := X^{(k)} + \omega A^T(B - AX^{(k)}), \quad k \geq 1$$

where the relaxation parameter ω verifies $0 < \omega < 2/\sigma_1^2$, being σ_1 the largest singular value of the transfer matrix.

- The *Range Restricted GMRES (RRGMRES)* [6]. The GMRES is based on the Krylov subspaces $K_k = \{d, Cd, C^2d, \cdots, C^kd\}$, $C = A^TA$, $D = A^Tb$, where the solution at iteration k is a vector in the k-dimensional subspace, $x^k \in K^k$, that minimizes the residual norm $||Cx^k - d||_2$. Then, GMRES at each iteration k minimizes the least squares problem:

$$x_k = \min_{z\in K_k} \|r_k - Cz\|_2$$

where r_k corresponds to the residual at iteration k. The method RRGMRES replaces the previous subspace by

$$\hat{K}_k = \{Cd, C^2d, \ldots, C^kd\}$$

since multiplying the noise component by the matrix C has a smoothing effect [6].

Iterative methods are semi-convergent, meaning that in the initial iterations, they improve their approximations to the solution until an iteration where they start to diverge due to the presence of noise and numerical errors

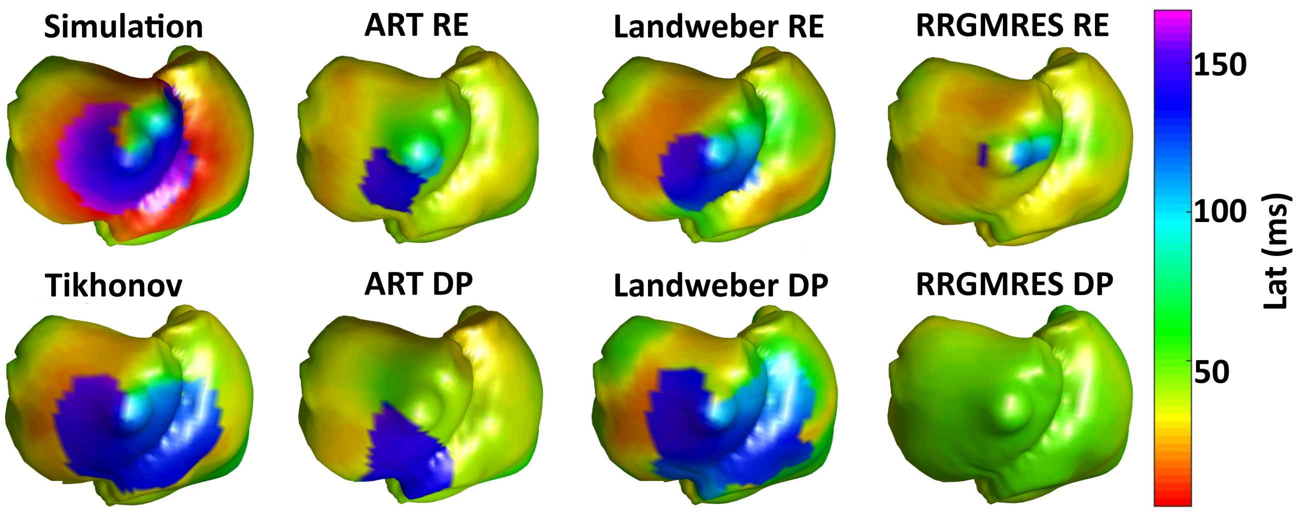

Figure 1. This figure shows the activation maps obtained with the different regularization methods, as well as the activation map obtained with the simulated pericardial potentials (EGM).

[6]. Therefore, an effective stopping criterion must be employed.
In this study, to evaluate the best performance of the iterative methods, we considered a (RE)-criterion that refers to the stopping criterion that stops the iterations when $||X^{(k)} - \hat{X}||_2 \geq ||X^{(k+1)} - \hat{X}||_2$, i.e., the iteration before the solution begins to diverge, where $\hat{X}$ refers to the exact solution on a fixed time instant.
Moreover, we also consider the Discrepancy Principle (DP)-criterion [11], a classical stopping criterion based on stopping the iterations when the norm of the residual is the same size as the norm of the noise on the right-hand side.

The efficacy of each computational approach is quantitatively assessed using the Correlation Coefficient (CC), computed for both the electrograms and the local activation times.

3. Results

We conducted simulations for five atrial ectopics and five atrial reentries to evaluate our methodologies. Our analyses, presented in Figure 1, showcase the activation maps generated using Zero Order Tikhonov regularization, the Landweber method, ART, and RRGMRES for a reentry simulation.
The correlation coefficient (CC) of the estimated electrograms indicated superior performance of the iterative methods with the RE criteria compared to Tikhonov regularization, as can be seen in Table 1. When focusing on the Local Activation Times (LATs), the iterative methods with the RE criteria obtain better mean results than Tikhonov regularization. However, Tikhonov regularization slightly outperformed the other methodologies for the reentry simulations. These findings are detailed in Table 2.

EGMs CC temporal	Ectopics	Reentries
Tikhonov	0.51 ± 0.03	0.26 ± 0.05
ART RE	0.52 ± 0.04	0.31 ± 0.06
Landweber RE	0.51 ± 0.06	0.35 ± 0.04
RRGMRES RE	0.50 ± 0.03	0.32 ± 0.05
ART DP	0.51 ± 0.02	0.26 ± 0.08
Landweber DP	0.50 ± 0.02	0.30 ± 0.05
RRGMRES DP	0.12 ± 0.01	0.01 ± 0.01

EGMs CC spatial	Ectopics	Reentries
Tikhonov	0.26 ± 0.05	0.56 ± 0.01
ART RE	0.31 ± 0.06	0.54 ± 0.01
Landweber RE	0.35 ± 0.04	0.57 ± 0.01
RRGMRES RE	0.32 ± 0.05	0.56 ± 0.01
ART DP	0.26 ± 0.08	0.25 ± 0.01
Landweber DP	0.30 ± 0.05	0.30 ± 0.04
RRGMRES DP	0.01 ± 0.01	0.01 ± 0.01

Table 1. These tables contain the CC temporal (top table) and spatial (bottom table) for the estimated EGMs with respect to the simulation.

4. Discussion and conclusions

This work provides a preliminary study on iterative methods for the estimation of electrograms for atrial ectopics and atrial reentries simulations. The iterative methods emerged as useful inverse methods with an effective stopping criteria, both estimating EGMs and LATs. Conversely, Tikhonov regularization with the L-curve obtained

LATs CC	Ectopics	Reentries
Tikhonov	0.64 ± 0.11	0.60 ± 0.05
ART RE	0.75 ± 0.07	0.56 ± 0.08
Landweber RE	0.71 ± 0.05	0.59 ± 0.38
RRGMRES RE	0.70 ± 0.06	0.53 ± 0.06
ART DP	0.74 ± 0.07	0.25 ± 0.15
Landweber DP	0.66 ± 0.16	0.34 ± 0.13
RRGMRES DP	0.11 ± 0.01	0.15 ± 0.02

Table 2. This table contain the CC for the local activation times with respect to the simulation

good results suggesting its continued relevance in ECGI.

Further work should explore the use of different stopping criteria for iterative methods, to study the practical achievement that these methods can obtain. Other future lines of research are the integration of these methods into clinical practice, and the development of hybrid or adaptive approaches that leverage the advantages of both methodologies.

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