Bayesian P Wave Amplitude Estimation

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Abstract

The P wave is an important feature of the electrocardiogram (ECG) because it can be used to diagnose atrial fibrillation and other arrhythmias. Most published P wave detection algorithms are merely estimate the time of the P wave, rather than the amplitude. We propose a simple algorithm for estimating the P wave amplitude. The algorithm is based on a parametric model that can be globally optimized, is computationally efficient, and only requires a single lead of the ECG. We used the QT Database for this study. The estimated P wave amplitudes had a Pearson correlation with the expert annotations of 0.94 ($p< 0.0001$).

1. Introduction

The QRS complex is the most prominent feature of the ECG, but the P wave is also important. Analysis of the P wave can be used to diagnose atrial fibrillation and other arrhythmias[1, 2]. It can also be used to distinguish between junctional rhythms and sinus rhythms, which is important for determining the need for a pacemaker.

Many previous groups have developed algorithms to detect and analyze properties of the P wave[3–6]. However, most of these algorithms are focused merely on detection of the P wave, and occasionally the P wave onset and offset. The performance of these algorithms is typically reported in terms of accuracy of the P wave detection and accuracy of the estimated onset, peak, and offset detection times. The accuracy of the P wave amplitude is rarely reported and is often not even estimated.

We propose a algorithm for estimating the P wave amplitude that overcomes these limitations. Estimating the P wave amplitude is more difficult than mere detection because the ECG waveforms are often noisy and the P wave is often superimposed on the T wave.

2. Database Selection and Annotations

For development and validation of the algorithm we selected the QT Database[7], which is available from Physionet[8]. This is a good database for this purpose because it contains a variety of ECG morphologies, some of which have poor signal quality. This makes the algorithm development much more challenging, but also ensures the algorithm will be applicable to a wide variety of applications and subject populations.

The QT Database contains 105 subjects with 15 minutes of 2-lead ECG recordings. The ECGs were recorded at 250 Hz with 12-bit resolution with a resolution of 10mV per arbitrary data unit (adu). The QT Database contains expert annotations of the P wave, QRS complex, and T wave. Most of the recordings were annotated by two experts. We used the annotations from the first expert (denoted as q1c) for our analysis. These annotations were not always accurate, and were not provided for all recordings.

Figure 1 shows an example of the P wave annotations in the QT Database (denoted as q1c). The annotations include the time of the P wave onset, peak, and offset shown by the lower olive triangle. Only the times are provided in the QT database, so we have drawn the amplitude using the original ECG signal, shown in gray. This shows the challenges of detecting P waves in this data set. The P wave contains two peaks and is quite noisy, which makes the amplitude estimates using the signal values inaccurate.

In the same figure a filtered ECG is shown by the blue trace. The filter design is described below. The filtered signal is slightly flatter and closer to the zero baseline. The signal is also much smoother. Both the times and amplitudes of the P wave onset, peak, and offset were annotated by our expert (SB) and are shown by the red triangle. The vertical line from the baseline to the annotated peak shows the P wave amplitude. As shown by this example, the new annotations are more accurate and provide a better standard to compare with our algorithm.

The number of annotated beats provided by the expert q1c varies across subjects. To facilitate statistical analysis and comparison between the q1c annotations and our expert annotations, we only annotated recordings that contained q1c annotations of at least 10 beats. Of the 105 subjects in the QT database, 97 subjects met this criterion. We also required that the ECG waveform not contain any clipping due to the signal exceeding the limits of

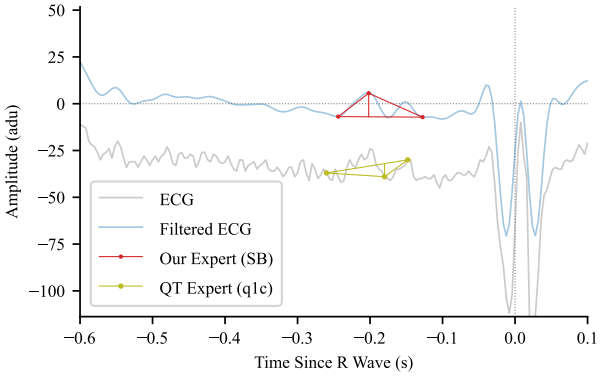

Figure 1: Annotated P wave from the QT Database. The olive triangle indicates the q1c annotation of the P wave onset, peak, and offset. The red triangle indicates the P wave onset, peak, and offset estimated by our expert (SB).

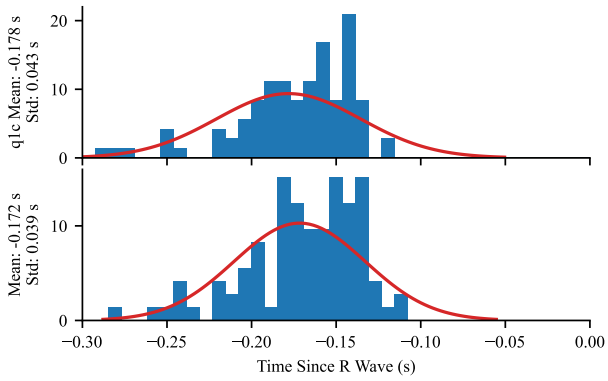

Figure 2: Histograms of the times of the P wave relative to the R wave as determined by the expert q1c (top) and our expert SB (bottom). The red curves show a Gaussian model of the distribution which has the same mean and standard deviation as the values estimated from the data.

the analog-to-digital converter. Only one subject (sel42) contained a clipped waveform. Finally, we required that the P wave have a non-negative amplitude, as determined by our expert. Only one subject (sele0129) had negative P wave amplitude. This resulted in a total of 95 subjects that were annotated by our expert, for a total of 950 annotated beats. Each annotation required approximately 10 s and the annotations were completed in approximately 3 h.

3. Algorithm Design

Our algorithm is designed to estimate the P wave amplitudes from a single lead ECG recording with detected R waves. There are many accurate algorithms for detecting R waves, so we did not develop a new algorithm for this purpose. Instead, we used the R wave annotations provided by the expert q1c.

ECG signals often contain significant baseline drift and noise. Typically a bandpass filter is used to remove these effects[9]. Keeping in mind that the P wave is usually smoother and broader than the QRS complex, for the purpose of detecting and estimating the P wave amplitude, it is possible to use a lower bandwidth than is typically used for QRS detection. We chose to use a Butterworth filter because it is simple and has no passband or stopband ripple. We chose the cutoff frequencies based on a visual inspection of the filtered P waves where we tried to eliminate as much baseline drift and high frequency noise as possible, without affecting the P wave amplitude. We found that a 4th-order Butterworth bandpass filter with a cutoff frequencies of 0.3 Hz and 30.0 Hz provided a good tradeoff between these competing objectives. Since this filter is applied to entire recordings, we filter the signal in both directions to avoid phase distortion. This effectively makes the filter an 8th-order filter.

Our model is only used to represent the portion of the ECG signal in which P waves typically occur. The P wave peak usually occurs 0.1 to 0.3 s before the R wave. Figure 2 shows the distribution of the times of the P wave relative to the R wave as determined by the q1c expert (top) and our expert (bottom). Accounting for the onset and offset of the P wave, we chose the modeled interval to range from the lesser of 0.33 s (t_{start}) and the inter-beat interval to 0.08 s (t_{end}) prior to the R wave. This ensured that the range always included the P wave, but for subjects with a high heart rate, this did not include the T wave from the previous beat.

A typical P wave visually resembles a bump on top of a smooth baseline. We chose to model this using a kernel function added to a simple quadratic model for the baseline:

$$\hat{x}(n;\alpha) = \alpha_0 + \alpha_1 n + \alpha_2 n^2 + \alpha_3 b(n;\alpha_4,\alpha_5) \qquad (1)$$

where n is the sample index, $\{\alpha_i\}_{i=0}^{5}$ are the six model parameters and $b(n;\alpha_4,\alpha_5)$ is the Biweight kernel function where α_4 is the time of the peak of the kernel and α_5 is the width of the kernel. This provides a simple way to represent the onset and offset of the P wave.

Our statistical model of the signal is $x(n) = \hat{x}(n)+v(n)$ where $v(n)$ is additive measurement noise. We assume the Central Limit Theorem (CLT) applies and therefore $v(n)$ can be modeled as a Gaussian random variable with zero mean and variance σ_v^2. We also assume the noise is white with zero mean, so $v(n)$ is uncorrelated for all n.

We used a Bayesian approach to estimate the model parameters because we had prior information that could improve algorithm accuracy. We chose to estimate the parameters by maximizing the posterior probability, as is typical with Bayesian estimators: $\hat{\alpha} = \arg\max_\alpha p(\alpha|x)$ where α

is the vector of model parameters $\{\alpha_i\}_{i=0}^{5}$, x is the vector of samples of $x(n)$ over the span of the P wave segment, and $p(\alpha|x)$ is the posterior probability density function of the model parameters given the P wave segment. The posterior probability density function is proportional to the product of the likelihood function and the prior probability density function: $p(\alpha|x) \propto p(x|\alpha)p(\alpha)$ where $p(x|\alpha)$ is the likelihood function and $p(a)$ is the prior probability density function of the model parameters. Since the measurement noise is Gaussian white noise with zero-mean, the likelihood function is given as follows (up to a constant of proportionality):

$$p(x|\alpha) \propto \exp\left(-\frac{1}{2\sigma_v^2}\sum_{n=0}^{N-1}\left(x(n)-\hat{x}(n;\alpha)\right)^2\right) \quad (2)$$

where σ_v^2 is the variance of the measurement noise. We only need to know the likelihood function up to a constant of proportionality since the solution is invariant to this constant.

We incorporated three types of prior knowledge in the design of the prior probability $p(\alpha)$. First, we know the P wave is non-negative. We accounted for this by using a prior $p_3(\alpha_3) = u(a_3)$ where $u(a)$ is the unit step function that is 1 for positive values and 0 for negative values.

Second, we know the width of the P wave is non-negative and typically spans the range of 0.1 to 0.7. Based on the CLT, we modeled the distribution of the P wave width as a Gaussian distribution with a mean of 0.1 s and a standard deviation of 0.025 s.

Finally, we know the P wave peak occurs sometime between 0.1 to 0.3 s before the R wave, as shown by the distribution of the annotated P wave peak times in Figure 2. This distribution is assymetric with no values above 0.1 s, so we modeled this as a trapazoidal distribution spanning 0.1 to 0.3 s prior to the R wave. This is constant from 0.25 to 0.12 s prior to the R wave, so it does not bias the estimate of the P wave peak time when it is within this range.

The model design was partially motivated by the desire to have a simple and efficient algorithm for estimating the model parameters that is not susceptible to local minima. Fortunately the model is linear in four of the six parameters, so these parameters can be estimated using linear least-squares estimation with a single linear constraint, $p(\alpha_3) \geq 0$. This can be solved optimally simply by estimating the model without the constraint using linear regression. If the estimated α_3 is negative, the constraint is violated, so we remove the kernel from the model and estimate the model without the kernel.

The remaining two parameters can be estimated by a simple grid search over the span of feasible values for the modeled P wave peak time and width. The grid search is performed by evaluating the likelihood function at each grid point and selecting the grid point with the maximum likelihood.

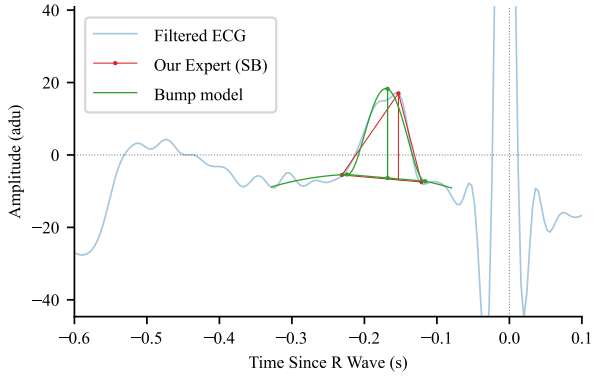

Figure 3: Example showing the modeled P wave segment (green curve) and the amplitude estimated from the model (vertical green line from baseline to peak). The filtered ECG is shown in blue, and the annotated P wave by our expert is shown in red.

Finally, the P wave amplitude is estimated as the difference between the baseline and the peak of the modeled P wave. The baseline was estimated in the same manner as the expert annotations described previously. Specifically, the onset was calculated as the greater of the start of the P wave segment and the start of the kernel. The offset was calculated as the lesser of the end of the P wave segment and the end of the kernel. The baseline was estimated as a straight line between these two points. The P wave amplitude was estimated as the difference between the baseline and the peak of the modeled P wave.

4. Results

Figure 3 shows an example of a modeled segment. Figure 4 shows a scatter plot of the estimated P wave amplitudes versus our expert annotations. The Pearson correlation coefficient was 0.96 ($p < 0.001$).

5. Discussion

While the proposed algorithm has excellent performance, it has several limitations. First, the algorithm is only designed for recordings with a non-negative P wave. Althought negative P waves are rare, they do occur. Second, the model uses a symmetric representation for the P wave, but the actual P waves often had a peak value that was not located at the midpoint between the onset and offset. The P waves also often contained two distinct peaks, rather than the single peak represented by our model. In these cases, the amplitudes may differ from the expert annotations. Third, the algorithm was both developed and

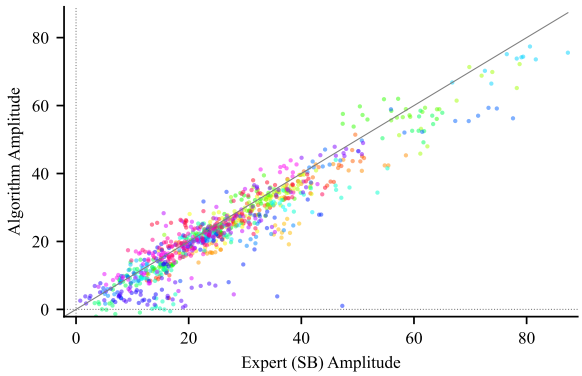

Figure 4: Scatter plot of the estimated P wave amplitudes versus our expert annotations. A different color is used for each subject. There are 10 beats for each of the 95 subjects for a total of 950 dots.

validated using the QT Database. This database contains a wide variety of ECG morphologies, but it is possible that some morphologies are not represented and that the algorithm would not perform as well on other datasets. Finally, our results are based on R waves annotated by an expert. In practical applications, the R wave detection will be performed by an algorithm, which may not be as accurate as an expert.

Our model assumed the additive noise was uncorrelated, but this is not an accurate assumption, partly because our bandpass filter introduces correlation. We don't believe this significantly affects performance, but it is possible to estimate the noise covariance matrix and incorporate this into the likelihood function for a small gain in accuracy at the expense of more computation.

Our annotations were completed by a single expert in a single session. We have not assessed either inter-expert or intra-expert variability to determine the reliability of the annotations.

6. Summary and Conclusion

We developed a simple algorithm for estimating the P wave amplitude. This algorithm is easy to understand, incorporates prior knowledge of properties of the P waves, and is globally optimized. It performed well on 950 beats from 95 subjects in the QT Database, with a Pearson correlation of 0.96 with expert annotations. We also developed a new set of expert annotations for the P wave onset, peak, and offset times and amplitudes for 950 P waves from 95 subjects in the QT Database. The algorithm was validated using these annotations.

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