Analysis of the Maximum A Posteriori Approach to the Inverse Problem of Electrocardiography for Different Depolarization Sequences

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Abstract

In this paper we investigate the previously proposed maximum a posteriori (MAP) approach to the problem of determining epicardial potentials from measured body surface potentials, a form of the inverse problem of electrocardiography. The MAP inverse approach uses a priori knowledge of the covariances between epicardial electrograms in its estimate of epicardial potentials. However, in practice, this information is not generally available. In this paper we examined the effectiveness of this method when the covariances are estimated using one depolarization sequence and the MAP method is used with these covariances to estimate the epicardial potentials for a different depolarization sequence.

1. Introduction

There have been many different approaches to the problem of estimating epicardial potentials from measured body surface potentials, a form of the inverse problem of electrocardiography. The most widely used methods are generally some form of Tikhonov regularization where the measured body surface potentials are matched to a model with an additional imposed penalty on either the magnitude or slope of the estimated potentials [1, 2, 3, 4, 3, 5, 6, 7, 8].

In this study we examined the maximum a posteriori (MAP) method of estimating epicardial potentials from measured body surface potentials which was proposed by van Oosterom [9]. This method assumes the covariances between epicardial electrograms are known in its estimate of epicardial potentials. However, in practice, this information is not generally known. In this paper we examined the effectiveness of this method when the covariances are estimated from one depolarization sequence and the MAP method is used with these covariances to estimate the epicardial potentials for a different depolarization sequence. The results for the MAP method are compared with those of standard zero order Tikhonov regularization.

2. Experimental data and model

For this paper, data was collected during an *in-vivo* experiment on swine. The swine model was used because of its similarity to humans in the anatomical arrangement of heart, lungs, bone, muscle, etc. For this experiment, bipolar pacing electrodes were sewn to the heart surface (the epicardium) in six different locations, and an epicardial sock was placed over the heart surface and over the pacing electrodes. The epicardial sock had effectively nine columns of six electrodes arranged about the heart. The chest was then sewn shut and unipolar recordings of epicardial potentials were made from the epicardial sock electrodes while the heart was paced from the six sites (Prucka Engineering, Houston, Texas).

A finite element model of the region from the epicardium to the torso was constructed using the I-DEAS finite element package (SDRC, Ohio) from CT scans made of the swine. For this study we assumed a homogeneous model with 41,430 nodes and 212,366 linear tetrahedral elements. The finite element model had 1748 nodes on the epicardium. In order to project the measured epicardial potentials at the 54 electrodes to the torso, we performed Laplacian interpolation from the measured electrodes to estimate the potentials at the remaining finite element nodes on the heart surface [10]. Using standard finite element techniques [3] a transfer matrix relating the (measured and estimated) epicardial potentials at all nodes on the heart surface to finite element nodes on the body surface was constructed. Specifically, our transfer matrix related the potentials at all 1748 epicardial nodes to 96 finite element nodes on the torso (body) surface which we assumed to be the measurement locations. These 96 torso surface locations were fairly evenly spaced near the heart on the pig torso.

The time segments analyzed consisted of the QRS

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portion of the cardiac cycle beginning just after the pacing spike. The RMS values of the measured epicardial potentials as a function of time for the portion of the QRS analyzed in this paper are displayed in Figure 1 for the six pacing sites. As the figure illustrates, the duration of the QRS varies as the source of the ventricular depolarization is varied, from a minimum of 75 milliseconds to 120 milliseconds.

In order to simulate modelling and measurement errors, white Gaussian noise was added to the computed body surface potentials. Once the body surface potentials were computed, the segments of time to be analyzed were determined and the corresponding body surface potentials were determined. The RMS value of the body surface potentials over the entire time period to be analyzed was determined as

$$RMS = \sqrt{\frac{1}{96N}\sum_{k=1}^{96}\sum_{i=1}^{N}(b_i^k)^2} \quad (1)$$

where N is the number of sample points in the QRS to be analyzed, b_i^k is the i^{th} sample point in the QRS at the k^{th} location. Once the RMS value was determined, zero mean white Gaussian noise with standard deviations of

$$\sigma = RMS * f \quad (2)$$

was added to the body surface potentials, where f was varied. Specifically, f was 0.05 for the 5% noise level used throughout this paper, although other levels of noise were also examined.

3. Inverse algorithms

In this section we introduce the two inverse algorithms we examined. Commonly used zero order Tikhonov regularization was used as a baseline and is introduced first, followed by the MAP method.

Zero Order Tikhonov Regularization. Zero order Tikhonov regularization can be formulated as the solution to the following minimization problem

$$\min_{\underline{\hat{h}}_i} \quad \Pi = ||\mathbf{Z}\underline{\hat{h}}_i - \underline{b}_i||^2 + \lambda_i||\underline{\hat{h}}_i||^2 \quad (3)$$

where $\underline{b}_i$ is a vector of known (measured) body surface potentials (at time i), $\mathbf{Z}$ is the transfer matrix, $\underline{\hat{h}}_i$ is the estimate of the potentials on the heart surface (the *epicardium*) surface (at time i) and λ_i is the regularization parameter (at time i). In general, we attempt to match the estimated body surface potentials ($\mathbf{Z}\underline{\hat{h}}_i$) to the measured body surface potentials, while penalizing epicardial estimates with large magnitude. The regularization parameter λ indicates the relative weight given to the two terms, and needs to be estimated based on measurable data. In this paper we estimated the regularization parameter using the Composite Residual Error and Smoothing Operator (CRESO) originally proposed by Colli-Franzone [5]. The estimated epicardial potentials at each time instant are then given as

$$\underline{\hat{h}}_i = (\mathbf{Z}^T\mathbf{Z} + \lambda_i\mathbf{I})^{-1}\mathbf{Z}^T\underline{b}_i \quad (4)$$

There is generally a different regularization parameter λ_i at each time instant.

Maximum A Posteriori Method. The epicardial potentials are estimated from the maximum a posteriori method (MAP) used in this paper as

$$\underline{\hat{h}}_i = \mathbf{\Gamma}_\Psi\mathbf{Z}^T(\mathbf{Z}\mathbf{\Gamma}_\Psi\mathbf{Z}^T + \mathbf{\Gamma}_\sigma)^{-1}\underline{b}_i \quad (5)$$

where $\mathbf{\Gamma}_\Psi$ is the estimated spatial covariance matrix of the epicardial potentials and $\mathbf{\Gamma}_\sigma$ is an estimate of the covariance matrix of the noise. For our simulations, we assumed we knew $\mathbf{\Gamma}_\sigma = \sigma^2\mathbf{I}$. The spatial covariance matrix was estimated as

$$\mathbf{\Gamma}_\Psi = \frac{1}{N-1}\sum_{i=1}^{N}(\underline{h}_i - \underline{\hat{\mu}})(\underline{h}_i - \underline{\hat{\mu}})^T \quad (6)$$

where

$$\underline{\hat{\mu}} = \frac{1}{N}\sum_{i=1}^{N}\underline{h}_i \quad (7)$$

is the estimated mean and N is the number of sample points in the depolarization sequence.

4. Results and discussion

Table 1 shows the average relative errors for each of the six depolarization sequences when standard zero order Tikhonov regularization is used to compute the epicardial estimate. This was considered the baseline result. Table 2 presents the average relative errors between the estimated and measured epicardial potentials when the correct covariances between epicardial potentials are known. Comparing these results to those presented in Table 1, it appears that the MAP method produces superior estimates to those produced by zero order Tikhonov regularization. However, in order to obtain these good results we need to know the correct covariances between the epicardial potentials. In essence, we need to know the answer before we start. Table 3 presents the average relative errors between the estimated and measured epicardial potentials when the wrong covariances are used with the MAP method for all of the protocols. Specifically, the first

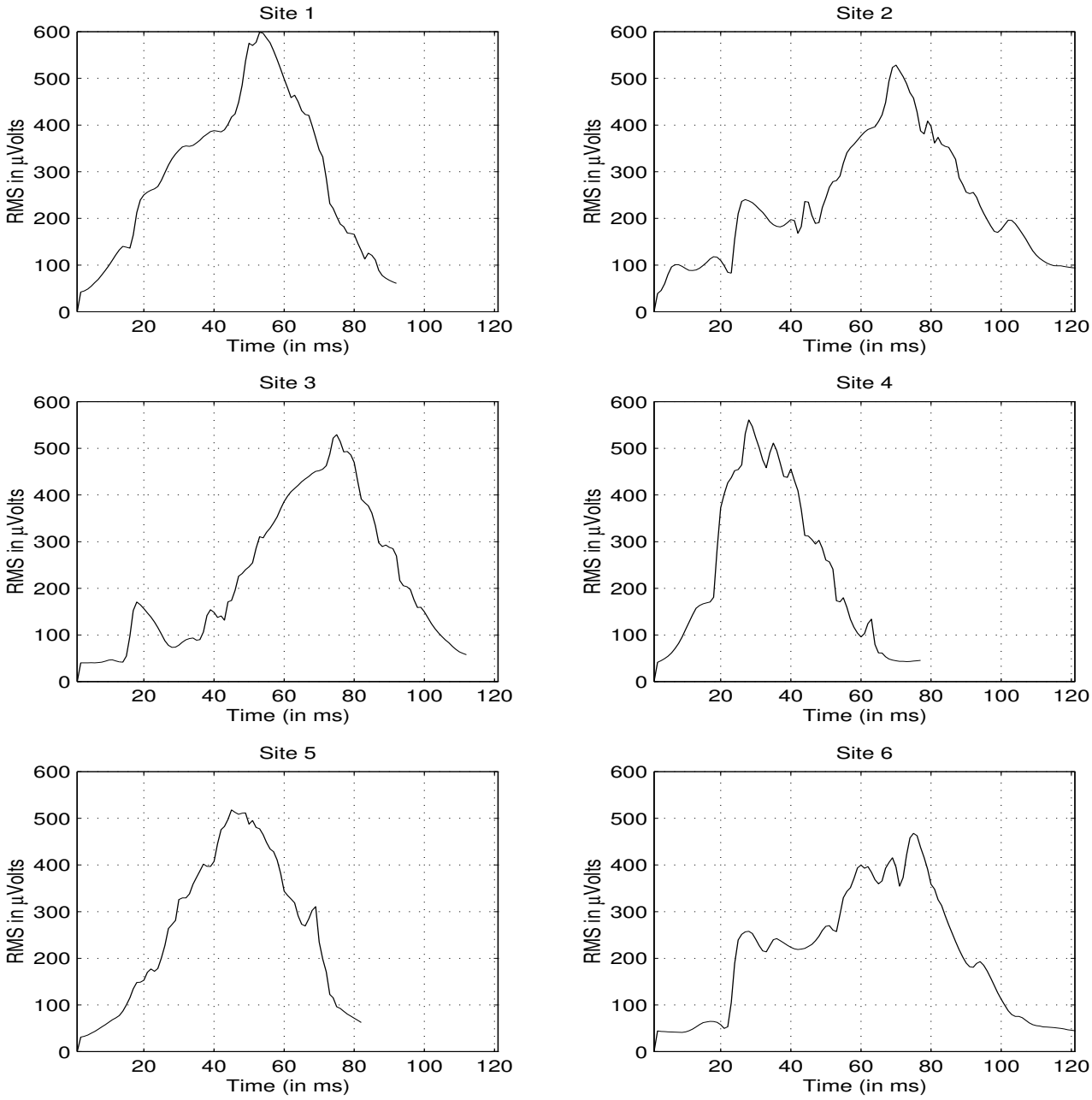

Figure 1. RMS values of measured epicardial potentials as a function of time for the six pacing sites. Each pacing site corresponded to a different depolarization sequence. The QRS duration ranged from 75 milliseconds to 120 milliseconds.

row shows the results when the covariances estimated from protocol zero are used with MAP to estimate the epicardial potentials for the other protocols. The second row shows the results when the covariances estimated from protocol one are used with MAP to estimate the epicardial potentials for the other protocols, and so on. As this table shows, for nearly all of the depolarization sequences, if the incorrect covariances are used, the epicardial estimates are worse than with standard zero order Tikhonov regularization.

	Site 1	Site 2	Site 3	Site 4	Site 5	Site 6
ave. RE	0.73	0.77	0.73	0.75	0.72	0.79

Table 1. Average relative errors between estimated and measured epicardial potentials using standard zero order Tikhonov regularization. There were six different pacing sites, one for each depolarization sequence.

	Site 1	Site 2	Site 3	Site 4	Site 5	Site 6
ave. RE	0.18	0.23	0.23	0.14	0.02	0.26

Table 2. Average relative errors between estimated and measured epicardial potentials when the correct covariances between epicardial potentials are known. There were six pacing sites, one for each depolarization sequence.

$\mathbf{\Gamma}_\Psi$ from	Site 1	Site 2	Site 3	Site 4	Site 5	Site 6
Site 1	–	1.83	1.55	1.40	1.10	1.85
Site 2	0.72	–	0.86	1.11	0.97	1.03
Site 3	0.78	0.84	–	0.90	0.92	0.83
Site 4	1.60	1.71	1.51	–	1.44	1.97
Site 5	1.74	1.69	1.53	1.46	–	1.49
Site 6	1.41	1.82	1.38	1.23	1.14	–

Table 3. Average relative errors between estimated and measured epicardial potentials when the covariances between epicardial potentials for the different depolarization sequence are used for other protocols. There were six pacing sites, one for each depolarization sequence.

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