The Use of the Simulation Results as a priori Information to Solve the Inverse Problem of Electrocardiography for a Patient

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Abstract

In the present work the epicardial potential distribution for an individual patient provided by the solution of the linear inverse problem of electrocardiography are shown. To obtain the solution, the Twomey regularization as well as the stochastic regularization were used. Both methods require a priori estimations. These estimations were provided by means of simulation of the cardiac activity on a personalized electrophysiological model of the patient.

To estimate the quality of the results provided by each method, the inverse problem was solved with the simulated ECG, the solution being compared with the epicardial potentials obtained from the simulation. Twomey regularization tends to provide the better correspondence than the conventional Tikhonov 0-order regularization. The stochastic regularization provides the best correspondence with the reference data, being the most time-consuming of all the methods under consideration.

Solving the inverse problem of electrocardiography provides a physician with the useful information about the electrophysiological processes in the heart of a patient.

1. Introduction

The inverse problem of electrocardiography is solved to get better insight into the cardiac processes without intervention. The problem is ill-posed, thus leading to large solution errors caused by arbitrary small errors in measurements and modeling.

In order to stabilize the solution, certain regularization methods must be applied. These methods require some a priori knowledge about the processes being reconstructed. In the most simple case, a limitation is posed on the amplitude of the solution (such as for the Tikhonov 0-order regularization) or its spatial derivatives (Tikhonov regularization of higher orders).

The Twomey regularization used in the present paper minimizes the difference between the solution and its a priori estimation. Back in 1992 Oster and Rudy used the epicardial potentials measured on a dog heart [1]. In this work this estimation is obtained on a virtual electrophysiological model of the cardiac activity of the patient described in [2].

The stochastic regularization estimates the inverse of a lead-field matrix based on the covariance matrices of the estimated solution as well as the covariance matrices of the measurement errors. These matrices were also computed from the simulation results.

Two different ECGs were used for the solution: the simulated ECG to control the output of each method (the epicardial potential distributions resulting from the simulation were used as the reference data) and the measured ECG of a patient.

2. Methods

Following steps were made to solve the inverse problem of electrocardiography. First, a personalized anatomical model of the patient was build from MRI-scans. Then, a cellular automaton was used to generate the transmembrane voltage distributions within the cardiac tissue for a single heart cycle. The forward problem of electrocardiography was solved to obtain the potential distributions within the whole volume conductor. The potentials on the surface of the heart were saved as a priori data; the body surface potentials formed the simulated ECG. The a priori information generated in this way was used for the Twomey regularization.

In order to build the covariance matrices for the stochastic regularization, the simulated epicardial potentials were saved for a number of heart cycles, each with different settings of the cellular automaton.

In following subsections these steps will be described in more detail.

2.1. Anatomical model

The anatomical model of the patient's thorax was built from the MR-images made on a Siemens Magnetom Vision tomograph ($B_z = 1.5T$) at the University Hospital in Würzburg, Germany. The scans have been segmented us-

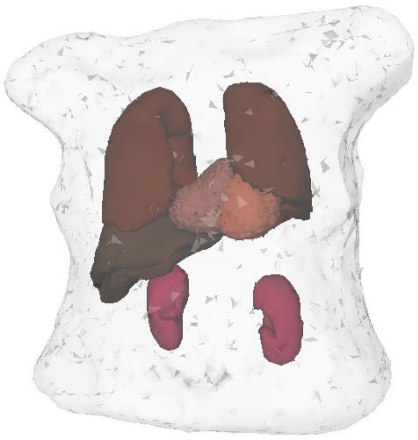

Figure 1. A simplified model of the volume conductor. Lungs, heart, liver and kidneys are shown.

ing the software developed at the Institute of Biomedical Engineering, University of Karlsruhe, Germany.

The resulting model was saved in voxel-based form (regular mesh) as well as in tetrahedron-based form (irregular mesh). This model contains following tissue classes: fat, heart, lungs, liver, kidneys, spleen and stomach (see Fig. 1).

2.2. Cellular automaton

The cellular automaton was run on a regular mesh containing the anatomical model of the patient's heart. This model was segmented from a short-axis MR-image with the resolution of 2.25x2.25x4 mm. The resulting mesh contains cubic voxels with the size of 1 mm. The model (see Fig. 2) contains following parts:

- Left and right ventricles;
- Blood inside the ventricles;
- Excitation propagation system (Purkinje fibers, Tawara bundles, fascicles) in the form of a sample tree starting from AV-node;
- Electrophysiological parameters such as action potential curves, tissue conductivities, excitation propagation velocities etc.

The excitation appears in AV-node, propagates through the excitation propagation system and reaches the ventricles. Action potential curves were computed from the ten Tusscher coupled cardiac cell model [3], the action potential heterogeneity through the ventricular wall was taken into account [4].

2.3. Forward problem

The transmembrane voltage distributions generated by the cellular automaton are interpolated to the irregular

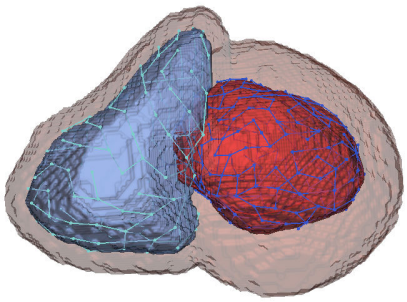

Figure 2. A heart model used by the cellular automaton containing ventricles, blood and excitation propagation system

mesh of the volume conductor model. Then the bidomain equations are solved to compute the cardiac current sources. Afterwards a Poisson equation is solved in order to obtain the potential distributions on the whole thorax [5].

2.4. Inverse problem

Following formulation of the inverse problem of electrocardiography was used. The epicardial potentials are connected with the potentials on the body surface at a given time instant with an equation:

$$A \cdot \boldsymbol{x} = \boldsymbol{b}, \tag{1}$$

where vector $\boldsymbol{x}$ means the potentials on the epicardial nodes of the volume conductor model, $\boldsymbol{b}$ is the corresponding body surface potential map (BSPM) and A designates the lead-field matrix. The task is to find the best possible $\boldsymbol{x}$ if $\boldsymbol{b}$ is given.

Matrix A is ill-conditioned, therefore its inverse cannot be computed directly. Minimum 2-norm pseudoinverse strongly amplifies the high-frequency noise and thus leads to the instability of the solution.

Most commonly used is the Tikhonov regularization damping the high-frequency part of the solution. It can be formulated as the following minimization problem:

$$\boldsymbol{x}_\lambda = \arg\min\left\{||A\boldsymbol{x} - \boldsymbol{b}||_2^2 + \lambda^2||L\boldsymbol{x}||_2^2\right\}. \tag{2}$$

Here L is termed the regularization matrix; the most frequent choice is the unit matrix. The regularization parameter λ controls the contribution of the regularization term $||L\boldsymbol{x}||_2^2$ into the functional being minimized [5].

The Twomey regularization being used in the current work includes the a priori estimation of the solution:

$$\boldsymbol{x}_\lambda = \arg\min\left\{||A\boldsymbol{x} - \boldsymbol{b}||_2^2 + \lambda^2||L(\boldsymbol{x} - \boldsymbol{x}^*)||_2^2\right\}, \tag{3}$$

here $\boldsymbol{x}^*$ designates the estimated solution.

Another approach used to find a pseudoinverse of A in (1) was described by David D. Jackson in [6]. An estimator H for the pseudoinverse is built:

$$H = C_x A^T (A C_x A^T + C_e)^{-1}, \quad (4)$$

here C_x is the covariance matrix describing the variability of $\boldsymbol{x}$, and C_e is the covariance matrix of the measurement errors.

To compute these matrices, the simulation process was run many times with the excitation propagation velocities being varied within the normal limits [7]. A pair of matrices C_x and C_e was calculated for each time instant.

3. Results

The inverse problem of electrocardiography was solved for both simulated and measured ECG. The results of the solution for the simulated ECG are shown in Fig. 3. A gaussian noise with $SNR = 100$ was added to the ECG in order to investigate the stability of the solution. The simulated epicardial potential distributions were used as reference.

The stochastic regularization (Fig. 3b) appeared to give the results being nearest to the reference. The epicardial potential distributions provided by this method are the most detailed, the smoothing typical for the Tikhonov regularization was in this case neglectably small. The drawback of this method is its calculation time: to compute the covariance matrices, one should run the simulations as many times as it is possible. In this case 81 simulations were held with 4 parameters being varied. Each simulation took about 5 min. on an Apple Xserve with 2 processors, 2 GHz each.

The smoothing effect caused by the Twomey regularization (Fig. 3c) appeared to be smaller than that specific to Tikhonov regularization (Fig. 3d). Still the improvement due to the introduction of a priori estimation was much smaller than in the previous method.

The reconstruction results obtained for the experimentally measured ECG are shown in Fig. 4.

4. Discussion and conclusions

In this paper only the spatial regularization was considered. Any BSPM at each time instance was treated separately, unlike the approaches described in [8]. Still the a priori estimation is changing continuously with time, therefore providing a dependence between the different time instances. In this way, the temporal regularization appears to be introduced indirectly.

Both presented methods deliver solutions with much more details visible in the images. Both methods rely on a priori knowledge about the solution. Twomey method comes with a stronger bias to the a priori estimate. The stochastic regularization is not so biased but demands for long calculation times.

Acknowledgements

This work was supported by the German Research Society (DFG).

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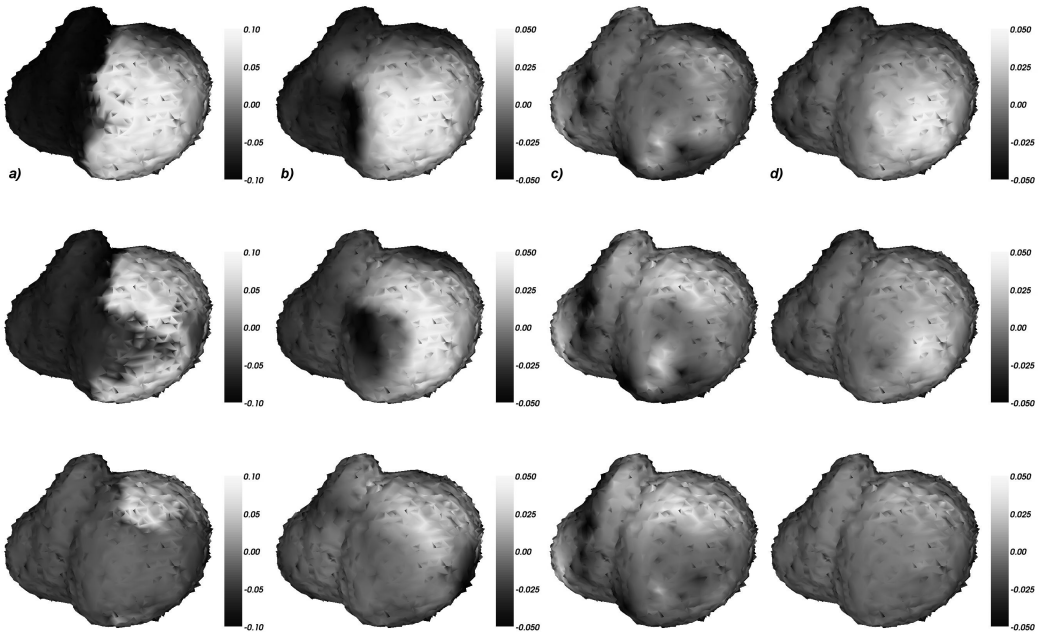

Figure 3. Epicardial potential distributions in 3 time instances reconstructed from the simulated ECG: a) reference, b) stochastic regularization, c) Twomey regularization ($\lambda = 1$), d) Tikhonov 0-order regularization ($\lambda = 0.03$). All values are given in V.

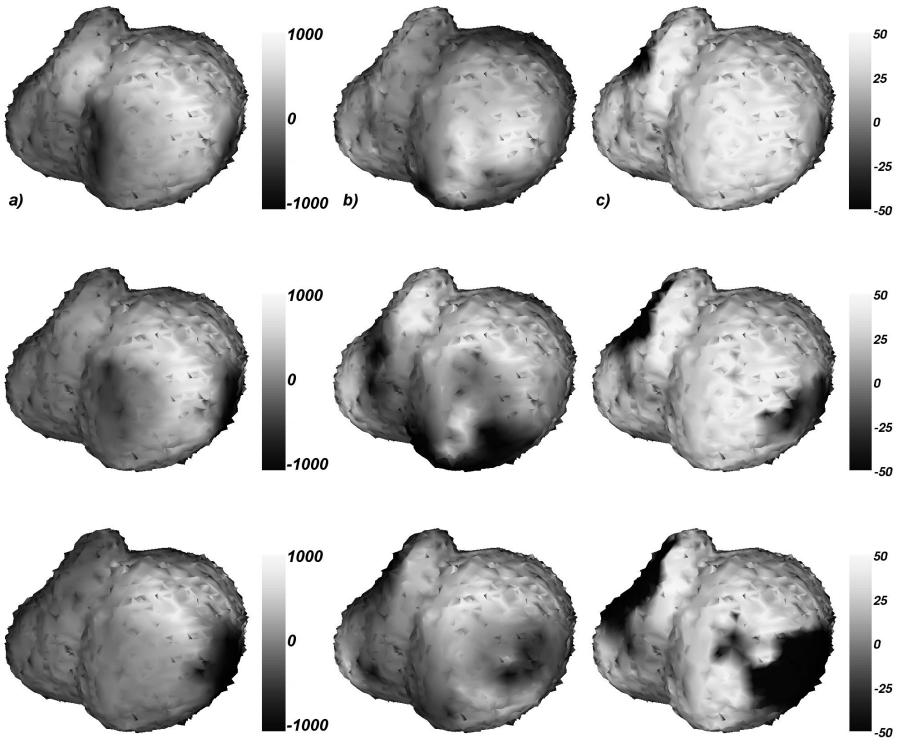

Figure 4. Epicardial potential distributions in 3 time instances reconstructed from the measured ECG: a) stochastic regularization, b) Twomey regularization ($\lambda = 1$), c) Tikhonov 0-order regularization ($\lambda = 0.03$). All values are given in a. u.