On the Use of ST-Segment Shifts and Mathematical Models for Identifying Ischemic Heart Disease

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Abstract

It is well known that myocardial ischemia can be observed as a shift in the ST segment of an ECG recording. In this paper we will discuss how ST-segment shift, along with advanced mathematical models, can be used to compute the size and location of the ischemic region - an inverse problem. Traditionally, the inverse approach to localizing ischemia has been based on the calculation of epicardial potentials. We present an alternative approach where the location and size of the ischemic region are computed directly by solving a parameter identification problem based on a level set framework and partial differential equations (PDEs). More precisely, the ischemic region is identified by minimizing an objective function measuring the difference between the recorded and simulated ECG signals. Our results show, at least for synthetic ECG data, that values of ST shift recorded at the body surface are capable of identifying the position and (roughly) the size of the ischemia.

1. Introduction

The bidomain equations, introduced by Tung [1] in the seventies, are widely accepted as an accurate model for the electrical activity in the myocardium. In this paper we will discuss how this model may be combined with biological insight, level set techniques, and modern laptops to derive a flexible framework for identifying ischemic heart disease.

Throughout the last three decades, computers have been extensively used to study the electrical activity in the human body. In particular, several scientists have analyzed how the epicardial potential can be determined from ECG recordings; see e.g. [2, 3]. This challenge may be formulated as a linear inverse problem for an elliptic PDE [4]. The use of such techniques, by transforming ST shifts measured at the body surface to the heart surface, for identifying infarctions have also been investigated [5, 6]. However, it is difficult to draw any final conclusion from this work, and whether or not such methods will come into clinical use is uncertain [7].

In this paper we will consider a somewhat different approach to this problem: instead of computing indirect indicators of ischemic heart disease, such as ST shifts, we aim at determining parameters directly describing the geometrical properties of the damaged tissue. This new approach leads to a nonlinear inverse problem for an elliptic PDE. In the Results section below, we will see, at least for synthetic observation data, that this framework is well-suited for this kind of application; by and large, our algorithm manages to identify the position and (roughly) the size of the ischemia.

A pilot study of the methodology discussed in this paper is presented in [8], and further details about the mathematical aspects of it can be found in [9]. The main purpose of the present work is to take these initial investigations one step further and test our ideas on a realistic 3D heart in a torso geometry.

2. Methods

Let H denote the domain occupied by the heart and consider the bidomain equations:

$$v_t + I(v,q) = \nabla\cdot(M_i\nabla v) + \nabla\cdot(M_i\nabla u) \text{ in } H, \quad (1)$$
$$\nabla\cdot(M_i\nabla v) + \nabla\cdot((M_i+M_e)\nabla u) = 0 \text{ in } H, \quad (2)$$

where v and u represent the transmembrane and extracellular potentials, respectively. The intra- and extracellular conductivity tensors M_i and M_e typically depend on the spatial position x, and the function I incorporates the ionic currents into the model. More specifically, I is a nonlinear function of both the transmembrane potential v and the ionic concentrations q; see [4] for details.

Let t_1 and t_2 be time instances during the plateau and resting states of the heart cycle, respectively. According to lab measurements

$$v(x,t_1) \approx \begin{cases} 0\text{mV} & x \text{ in healthy tissue,} \\ -20\text{mV} & x \text{ in ischemic tissue,} \end{cases} \quad (3)$$

and

$$v(x,t_2) \approx \begin{cases} -90\text{mV} & x \text{ in healthy tissue,} \\ -60\text{mV} & x \text{ in ischemic tissue;} \end{cases} \quad (4)$$

see [10]. From (3) and (4) we conclude that the shift in the membrane potential v is

$$\begin{aligned} h(x) &= v(x,t_1) - v(x,t_2) \\ &\approx \begin{cases} 90\text{mV} & x \text{ in healthy tissue,} \\ 40\text{mV} & x \text{ in ischemic tissue.} \end{cases} \end{aligned} \quad (5)$$

Moreover, the linearity of (2) implies that the ST shift $s(x) = u(x,t_1) - u(x,t_2)$ in the potential u must obey the equation

$$\nabla \cdot ((M_i + M_e)\nabla s) = -\nabla \cdot (M_i \nabla h). \quad (6)$$

Let G denote the Heaviside function; $G(s) = 0$ for $s < 0$ and $G(s) = 1$ for $s \geq 0$. If ϕ is a level set function satisfying

$$\begin{aligned} &\phi(x) < 0 \quad \text{if } x \text{ is in ischemic tissue,} \\ &\phi(x) = 0 \quad \text{if } x \text{ is at the border of ischemic tissue,} \\ &\phi(x) > 0 \quad \text{if } x \text{ is in healthy tissue,} \end{aligned}$$

then we may express h in (5) in the form

$$h(\phi) = 40(1 - G(\phi)) + 90G(\phi). \quad (7)$$

Consequently, if the position of the ischemia is known, then we can easily incorporate the effect of this disease on the simulated ST shift on the body surface by applying formula (7) and solving (6) for s. Furthermore, from such a perspective, it is natural to consider s to be a function of ϕ, $s = s(\phi)$. In the present context, the task of computing $s(\phi)$ is the direct problem. However, we are mainly concerned with its inverse counterpart - to determine the geometrical properties of the ischemia from body surface measurements of the ST shift.

Note that if ϕ is known then we can easily determine the position and size of the damaged tissue from the set $\{x;\, \phi(x) < 0\}$. Thus, we aim at developing a suitable framework for computing ϕ. To this end, let d denote the recorded ST shift at the leads positioned at the body surface, i.e. d is our observation data. The main idea is to try to determine ϕ such that the deviation between the simulated and observed ST shifts is as small as possible, i.e. we want to solve the minimization problem

$$\min_{\phi} \sum_{\text{all leads}} (s(\phi) - d)^2, \quad (8)$$

subject to $s = s(\phi)$ satisfying (6).

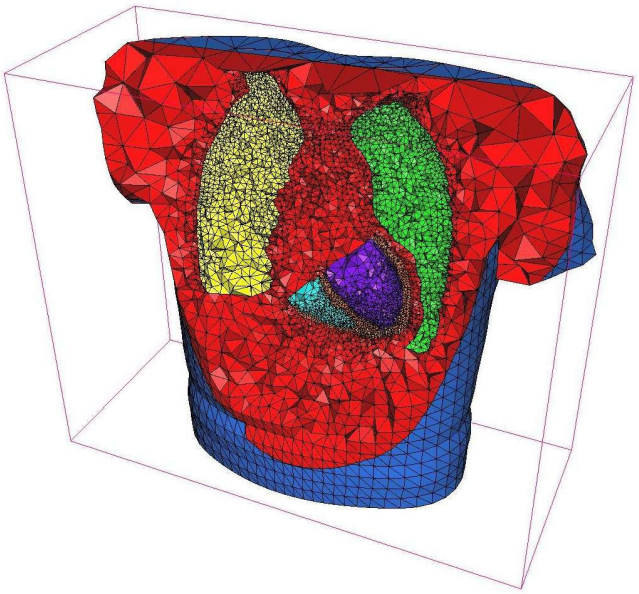


Figure 1. A tetrahedrazation of the human torso obtained from MR images.

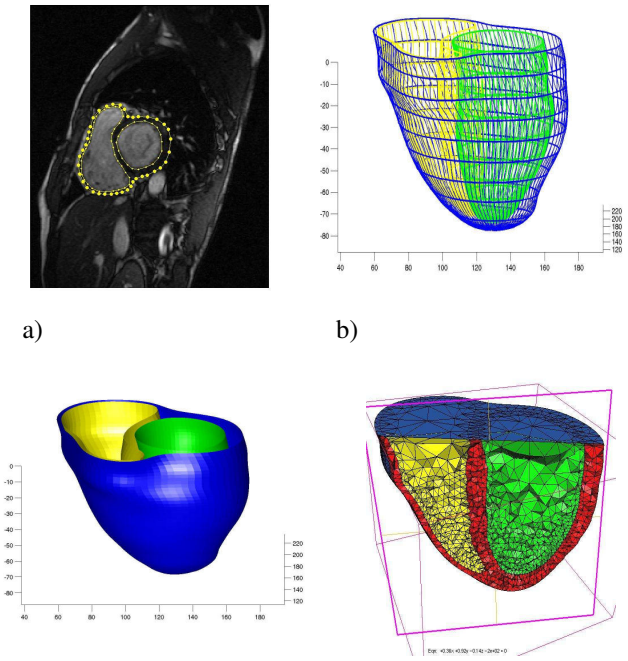
In the experiments presented below, equation (6), including the level set function ϕ, was discretized by the finite element method, and (8) was solved with a descent scheme; further details can be found in [9].

The discretization of (6) required by the finite element method involves the computation of a tetrahedrazation of the domain in question. In our case, the domain is the human torso and the tetrahedrazation is constructed from MR images of a real person. The MR images are delivered in two chunks recorded from different angles:

- The *body images* are recorded perpendicularly to the longitudinal axis of the body. The distance between the images is 1.5 cm.
- The *heart images* are recorded perpendicularly to the longitudinal axis of the heart. The distance between the images is 1.0 cm.

The final torso model, see Figure 1, contains the heart with the ventricles and the lungs.

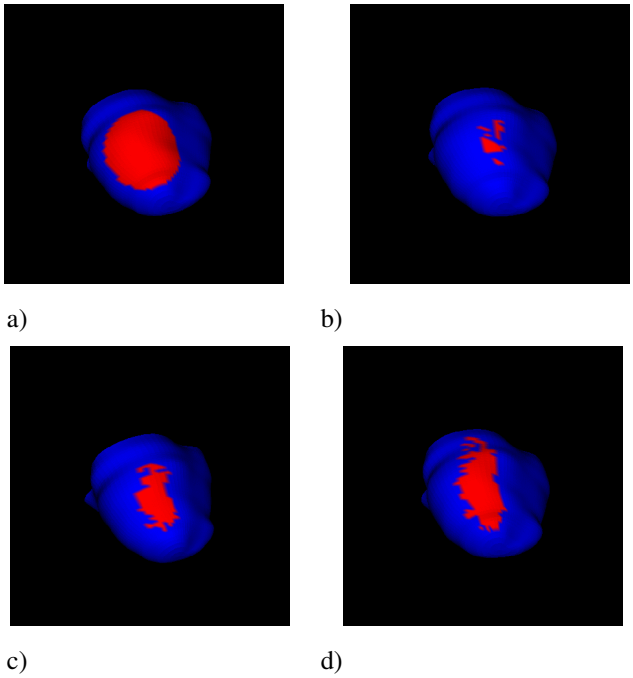
The first main step in our approach consists in making a smooth surface model. (This gives us a way of improving the resolution in the data). For each part (body, heart, ventricles, lungs) a closed surface is constructed. Thereafter, they are assembled into a surface model containing all parts. Finally, this surface model is turned into a tetrahedrazation by the software package called *TetMesh-GHS3D*, commercialized by the company called Distene (www.distene.com). The procedure can be summarized as follows:



a) b)

c) d)

Figure 2. The various steps in the procedure for constructing a 3D simulation grid from MR images.



a) b)

c) d)

Figure 3. The posterior ischemia we try to identify is shown in a). Figures b), c) and d) contain the results obtained after 4, 8 and 12 iterations by our minimization scheme, respectively.

1. *Surface model generation*
   (a) *Manual segmentation into curves*
   (b) *Determine correspondence between data curves*
   (c) *Construct spline curves interpolating data curves*
   (d) *Generate a surface triangulation*
2. *Tetrahedrazation*

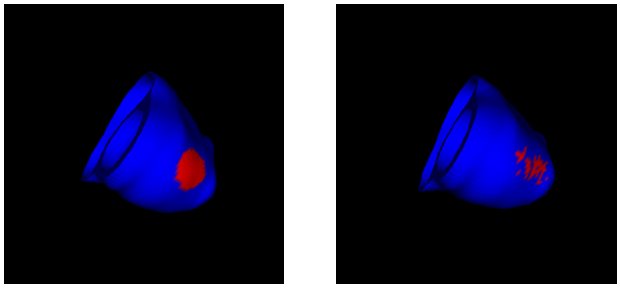
Figure 2 shows the different steps of the surface modeling procedure when applied to the heart only. The first step is the segmentation of the relevant body parts by manual drawing of, what we call, the *data curves*, see Figure 2 a). Next, part 1 b) and c) in the procedure result in a network of spline curves connecting the segmented data curves. This technique is called lofting (see [11]) and is visualized in Figure 2 b). In the next step, a surface triangulation is constructed from this curve network, see Figure 2 c). The final tetrahedrazation is shown in Figure 2 d).

In order to assemble the complete torso model shown in Figure 1, we need to take into account that the heart and torso images were recorded from different angles. The heart must consequently be transformed into the coordinate system of the torso data. The machine producing the MR images provides us with *some* of the required information for doing this. Rotational information together with scaling is stored but translational information is lost. This means that the orientation and size of the heart is known. However, the actual positioning of the heart was done by hand. As a guide in this process, we computed cross-sectional images of the assembled model and compared them with the torso data images by visual inspection.

3. Results

In this section we present some of the results obtained with our scheme for the real heart in torso geometry described above. In the first numerical example we try to identify a posterior circular ischemia as shown in Figure 3 a). In the ischemic region $\phi < 0$, while $\phi > 0$ in the rest of the heart H. To generate synthetic ECG boundary measurements d, we solve (6)-(7) for the given artificial ischemia. All knowledge of ϕ is thereafter put aside, and we try to recover the ischemia by only using the observation data d. As an initial guess for our algorithm we assume that no ischemia is present, i.e. $\phi > 0$ throughout H. In Figure 3 we show the estimated ischemia generated by an increasing number of iterations of the descent method used to solve (8).

After 4 iterations, $\phi < 0$ in some regions in H, as shown in Figure 3 b). Already at this early stage the position of the ischemia is identified rather accurately with respect to the center of mass; the relative error in the computed center of mass is approximately 8.3% (corresponding to 0.56cm). However, the size is obviously underestimated. As the



a) b)

Figure 4. The posterior ischemia we try to identify is shown in a), while our optimal estimate of the ischemia is shown in b).

number of iterations increase, the size of the proposed ischemic region evolves, while the center of mass remains more or less unchanged. After 12 iterations the characteristics of the ischemia did not change anymore, and the algorithm was stopped. By comparing the results in figures 3 a) and d) we conclude that the position, i.e. the center of mass, of the ischemia is estimated quite well, while the size of the ischemia is somewhat underestimated.

In the next example we try to identify a smaller ischemia, see Figure 4 a). Please note that the position of the ischemic region is changed as well, compared with the previous example. Our result is visualized in Figure 4.

Also in this test the algorithm was stopped after 12 iterations. By comparing the images in figures 4 a) and b) we see that our solution consists of several unconnected small regions, while there in fact should have been just one region. However, these small regions are all nicely placed at the right position. In this case the relative error in the estimated center of mass is 12.5% (corresponding to 0.83cm).

4. Discussion and conclusions

In this paper we have explored the possibilities for using measurements of the electrical potential at the surface of the body, mathematical techniques and computers to determine the position and size of a myocardial ischemia. Our framework is based on the bidomain equations, modern level set methods and geometrical tools suitable for construction high resolution meshes from MR images. Following the theoretical considerations, presented in Section 2, a series of experiments were presented in Section 3.

Our results are promising. For synthetic ECG data, the solution of the minimization problem (8) indeed holds significant information about the geometrical characteristics of the ischemia. And, it is possible to generate high quality grids, with a rather limited amount of manual labor, from MR recordings. Nevertheless, it is not clear whether, or to what extent, such techniques will/can come into clinical use: Tests on real world data should be performed, and both the mathematical and computational issues must be further developed.

Acknowledgments

It is a pleasure to thank Glenn Terje Lines and Joakim Sundnes for several interesting discussions on C++ programming and electrophysiology!

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