Time-Frequency Analysis of Heart Rate Variability during Stress Testing Using "a Priori" Information of Respiratory Frequency

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Abstract

A parametric decomposition of the smoothed Wigner-Ville distribution is applied to estimate the instantaneous frequency and amplitude of the LF and HF components of HRV during stress testing. It is assumed that the instantaneous frequencies of the LF and HF components may vary linearly with time during stress testing, and that the frequency of the HF component can be approximated by the respiratory frequency. The effect of the inclusion of "a priori" information of respiratory frequency on the estimation of HRV parameters is studied. Results on a simulation study show that the inclusion of "a priori" information does not affect significantly the estimation of the LF parameters. However, the SD of the estimation error of the HF amplitude is reduced at the expense of introducing a bias for high SNRs while both the mean and SD of the HF amplitude error are decreased for lower SNRs (2.6%±1.9% against 3.1%±2.6% for a SNR of 20 dB and 7.6%±5.8% against 12.0%±9.7% for a SNR of 10 dB).

1. Introduction

Time-frequency methods have been extensively applied in the study of non-stationary heart rate variability (HRV). That is the case of the non-parametric quadratic time-frequency distributions, such as the Wigner-Ville distribution (WD) and every filtered version of the WD, known as the Cohen's class.

An automatic decomposition of the smoothed Wigner-Ville distribution (SWD) has been applied to estimate the instantaneous frequency and power of the LF and HF components of HRV during tilt test [1]. The method performs a parametric decomposition of the instantaneous autocorrelation function (ACF) of HRV, based on the fact that the ACF of a signal composed of complex sinusoids whose instantaneous frequencies may vary linearly in time, can be decomposed at each time instant as a sum of complex sinusoids.

During stress testing HRV is highly non-stationary and it can be modeled as a sum of sinusoids with linearly varying instantaneous frequencies. The frequency of the LF component can be assumed constant during the test while the frequency of the HF component can be assumed to increase linearly from the beginning of the test to the stress peak and decrease linearly in the recovery phase. It is widely accepted that the HF component of HRV is driven by the action of the parasympathetic branch of the autonomic nervous system and that it is affected by respiration [2]. Based on this physiological knowledge and experimental observations, the frequency of the HF component can be approximated by the respiratory frequency, which may be derived directly from simultaneously recorded respiratory signals or indirectly from the ECG [3].

The objective of this work is twofold: first, to evaluate the performance of the method proposed in [1] to estimate the instantaneous frequency and power of the LF and HF components of HRV during stress testing and, second, to study the effect of the inclusion of "a priori" knowledge, such as the respiratory frequency, on the estimation of HRV parameters.

2. Methods and materials

2.1. The SWD and the ACF

The smoothed windowed discrete Wigner-Ville distribution of the discrete signal $x(n)$ is defined by [4]

$$X(n,m)=2\sum_{k=-N+1}^{N-1}|h(k)|^2\sum_{p=-M+1}^{M-1}g(p)r_x(n+p,k)e^{-j2\pi km/N}, \tag{1}$$

where n and m are the discrete time and frequency indexes, respectively, $h(k)$ is the frequency smoothing symmetric normed window of length $2N-1$, $g(p)$ is the time smoothing symmetric normed window of length $2M-1$ and $r_x(n,k)$ is the instantaneous ACF defined as $r_x(n,k)=x(n+k)x^*(n-k)$. The SWD can be seen as the Fourier transform of the filtered ACF

$$s_x(n,k)=|h(k)|^2\sum_{p=-M+1}^{M-1}g(p)r_x(n+p,k). \tag{2}$$

The method proposed in [1] is based on the fact that if $x(n)$ is composed of complex sinusoids whose instantaneous frequencies may vary linearly in time, then $s_x(n,k)$ can be approximated by a sum of complex damped sinusoids, corresponding to both signal and interference terms,

$$s_x(n,k) \simeq \sum_{i=1}^{I} c_i(n) e^{-b_i(n)k} e^{\jmath\omega_i(n)k}, \quad k=0,\ldots,N-1, \tag{3}$$

where $c_i(n)$ denotes the amplitude, $b_i(n)$ the damping factor and $\omega_i(n)$ the angular frequency of the i^{th} term.

The analytic signal of the HRV during stress testing $x(n)$ is assumed to be composed of two complex sinusoids: one with constant instantaneous frequency f_{LF} and amplitude A_{LF}, representing the LF component, and the other one with linearly varying instantaneous frequency $f_{\text{HF}}(n) = 2\alpha n + \beta$ and amplitude A_{HF}, representing the HF component,

$$x(n) = A_{\text{LF}} e^{\jmath 2\pi f_{\text{LF}} n} + A_{\text{HF}} e^{\jmath 2\pi(\alpha n^2 + \beta n)}. \tag{4}$$

If a rectangular window $g(p)$ of length $2M-1$ and an exponential window $|h(k)|^2 = e^{-\gamma|k|}$ are used[1], then

$$\begin{aligned} s_x(n,k) &= |A_{\text{LF}}|^2 e^{-\gamma|k|} e^{\jmath 2\pi f_{\text{LF}} 2k} + \\ &\tfrac{1}{2M-1}|A_{\text{HF}}|^2 e^{-\gamma|k|} \tfrac{\sin(2\pi 2\alpha(2M-1)k)}{\sin(2\pi 2\alpha k)} e^{\jmath 2\pi(2\alpha n+\beta)2k} + \\ &\tfrac{1}{2M-1} 2\Re\{A_{\text{LF}} A_{\text{HF}}^*\} e^{-\gamma|k|} e^{\jmath 2\pi(f_{\text{LF}}+f_{\text{HF}}(n))k} \,. \\ &\sum_{p=-M+1}^{M-1} \left\{\cos[2\pi\{\alpha[(n+p)^2+k^2]+(\beta-f_{\text{LF}})(n+p)\}] e^{\jmath 2\pi 2\alpha pk}\right\}, \end{aligned} \tag{5}$$

which can be approximated as in (3). The term related to the HF component can be approximated as a damped complex sinusoid since it has a varying amplitude, which can be approximated by an exponentially decaying function of k. The amplitude of the interference term, i.e. the last term in (5), is supposed to be largely reduced by the time smoothing and it will be treated as noise.

2.2. Estimation of the parameters

The parameters of the complex damped sinusoids of $s_x(n,k)$ in (3) can be estimated from a LS linear prediction problem and a singular value decomposition to make the estimation robust against noise [5]. The parameters $b_i(n)$ and $\omega_i(n)$ are estimated from the zeros of the prediction error filter polynomial of order L, $z_i(n) = e^{-s_i^*(n)}$, with $s_i(n) = -b_i(n) + \jmath\omega_i(n)$, and $c_i(n)$ as the LS solution to (3). Then, the frequencies and amplitudes of the complex sinusoids of $x(n)$ are estimated as

[1] The time smoothing window $g(p)$ should be chosen as a compromise between reducing the amplitude of the interference term while following the non-stationary characteristics of $x(n)$. The frequency smoothing window $h(k)$ should be chosen so as to increase frequency resolution while ensuring that $s_x(n,k)$ can still be modeled as in (3).

$$\hat{f}_i(n) = \frac{1}{2}\frac{\hat{\omega}_i(n)}{2\pi}, \quad \hat{A}_i(n) = \sqrt{\hat{c}_i(n)}. \tag{6}$$

The LF and HF components are identified as the sinusoid with highest power whose estimated frequency $\hat{f}_i$ lies in the band [0.04,0.15] Hz and [0.15,HR/2] Hz, respectively.

2.3. Inclusion of "a priori" information

It is generally accepted that the HF component of HRV reflects respiratory sinus arrhythmia [2]. This component is synchronous with the respiratory frequency. Then, "a priori" information about respiration can be included in the estimation of the HRV components. If the zero z_{HF} associated to the HF component is known, the estimation of the prediction error filter polynomial can be solved as a constrained LS problem. The knowledge of the zero $z_{\text{HF}} = e^{b_{\text{HF}}+\jmath 2\pi 2 f_{\text{HF}}(n)}$ requires the knowledge not only of the instantaneous frequency $f_{\text{HF}}(n)$, which is approximated by the respiratory frequency, but of the damping factor b_{HF}. One simple approach is to approximate the envelope of the function $m(k) = \frac{1}{2M-1}\frac{\sin(2\pi 2\alpha(2M-1)k)}{\sin(2\pi 2\alpha k)}$ by an exponential fit $e(k) = e^{-\delta|k|}$, then, the damping factor b_{HF} can be approximated by $b_{\text{HF}} = \gamma + \delta$. As the function $m(k)$ is periodic with period π, the approximation is only valid for $2\pi 2\alpha k < \frac{\pi}{2}$, i.e. $8\alpha k < 1$. Due to the exponential window $|h(k)|^2 = e^{-\gamma|k|}$, the first samples of $m(k)$ are those most influencing the parameter estimation. The first samples of $m(k)$ are approximated by a LS exponential fit $e(k) = e^{-\delta|k|}$, which can also be approximated by a linear fit $e(k) \simeq 1 - \delta|k|$ for $|k|\delta \ll 1$, giving a value of δ which is function of 2α, i.e. the rate of variation of $f_{\text{HF}}(n)$.

2.4. Time varying smoothing window

The amplitude and bandwidth of the spectral peak corresponding to the HF component is dependent on the rate of variation of $f_{\text{HF}}(n)$, 2α, and on the length of the time smoothing window $2M-1$. As a result, for a fixed time smoothing window length $2M-1$, the estimation error of A_{HF} depends on the rate of variation of its frequency (2α), being larger for faster variations. In order to diminish the differences between estimation errors of A_{HF} for different values of 2α, a time varying SWD can be applied, in which the time smoothing window length $2M-1$ changes with the value of 2α, so that $\frac{\sin(2\pi 2\alpha_1(2M_1-1)k)}{\sin(2\pi 2\alpha_1 k)} = \frac{\sin(2\pi 2\alpha_2(2M_2-1)k)}{\sin(2\pi 2\alpha_2 k)}$. If $\sin(2\pi 2\alpha_1 k) \simeq \sin(2\pi 2\alpha_2 k) \simeq 1$, the former relation can be written as $\frac{2\alpha 1}{2\alpha_2} = \frac{2M_2-1}{2M_1-1}$.

2.5. Simulation study

A simulated HRV signal is generated such that its analytic function $x(n)$ has the form of (4). The following parameter values are used: $A_{\text{LF}} = 1$, $f_{\text{LF}} = 0.1Hz$, $A_{\text{HF}} = 1$,

$$f_{\text{HF}}(n) = \begin{cases} 2\alpha_1 n + \beta_1 & t_o \le n \le t_p \\ 2\alpha_2 n + \beta_2 & t_p < n \le t_e \end{cases} \quad (7)$$

where $\alpha_1 = \frac{1}{3000}$ Hz/s, $\beta_1 = 0.25$ Hz, $\alpha_2 = -\frac{1}{1000}$ Hz/s, $\beta_2 = 2.05$ Hz, $t_o = 0$ s, $t_e = 900$ s and $t_p = 0.75(t_e - t_o) + t_o$ s. The sampling frequency is set to $f_s = 4$ Hz. The values of the parameters f_{LF}, $f_{\text{HF}}(n)$, t_o, t_e and t_p are selected based on the observation of actual HRV signals during stress testing, using rates of variation of $f_{\text{HF}}(n)$, i.e. α_1 and α_2, always steeper than observed in reality [3].

3. Results

The time varying SWD is applied to the simulated signal $x(n)$ using a time varying rectangular window $g(p)$ (with $2\alpha_i(2M_i - 1) = \frac{1}{120}$ Hz, $i = 1, 2$) and an exponential window $h(k)$ of length $2N - 1 = 256$ samples (64 s) and damping factor $\gamma = \frac{1}{64}$ samples^{-1} (16 s^{-1}). It is displayed in Fig. 1, where it can be appreciated that the amplitude and bandwidth of the spectral peak of the HF component are approximately the same independently of the value of 2α. The respiratory frequency is supposed to be known and to be exactly equal to $f_{\text{HF}}(n)$. The first $\frac{2N-1}{4}$ samples of $m(k)$ are used to estimate the damping factor b_{HF}.

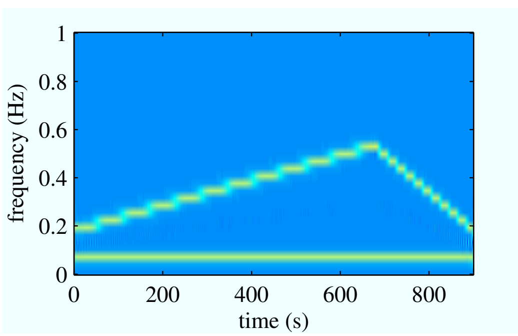

Figure 1. The time varying SWD of the simulated $x(n)$ using adaptive time smoothing window length.

The inclusion of "a priori" information of respiratory frequency does not significantly affect the estimation of the LF parameters, $\hat{f}_{\text{LF}}$ and $\hat{A}_{\text{LF}}$. However, the variance of the estimated HF amplitude $\hat{A}_{\text{HF}}$ is considerably reduced at the expense of increasing the bias, as it can be seen in Fig. 2(top). Outlier estimates are observed in the vicinity of time instant t_p, where 2α changes and the assumption of components with linearly varying instantaneous frequencies does not hold.

Estimation errors for the amplitude and frequency of the LF and HF components are computed when white gaussian noise is added to the simulated HRV signal for different values of the order L and different signal-to-noise ratios (SNR, defined as the ratio between A_{LF} and the standard deviation (SD) of the noise). Figure 2(bottom) displays $\hat{A}_{\text{HF}}$ for a SNR of 20 dB and averaged among 100 realizations, estimated with and without inclusion of "a priori" information of respiratory frequency.

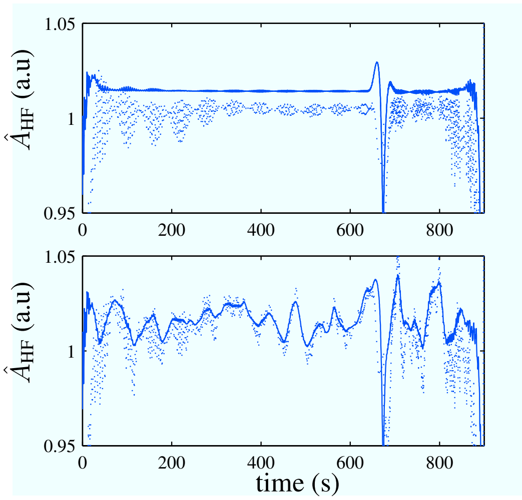

Figure 2. Amplitude of the HF component $\hat{A}_{\text{HF}}$ (in arbitrary units, a.u.) estimated by the method with (solid line) and without (dotted line) inclusion of "a priori" information of respiratory frequency using L=12 for an infinity SNR (top) and for a SNR of 20 dB (bottom).

Mean and SD of the estimation errors over time are computed and averaged among 100 realizations, excluding the beginning, the end and the vicinity of t_p, where assumptions do not hold. The LF parameters are estimated accurately both by the method in 2.2 and in 2.3. The f_{LF} is underestimated with a relative error lower than -0.6%±1.8% (mean±SD) for values of L between 10 and 20 and SNRs as low as 10 dB. The mean estimation error of A_{LF} is below 0.5% for SNRs above 20 dB, although its SD increases with decreasing SNR up to 4%. The estimation of f_{HF} without "a priori" information is accurate (estimation error below 0.5%±0.5%) up to SNRs of 15 dB. The estimation of A_{HF} is the most affected by the inclusion of "a priori" information of respiratory frequency. The mean and the SD of the absolute estimation error of A_{HF}, $m_{|\Delta_{A_{\text{HF}}}|}$ and $\sigma_{|\Delta_{A_{\text{HF}}}|}$, are displayed as a function of L for a SNR of 20 dB in Fig. 3, and as a function of the SNR for a value of L=12 in Fig. 4.

For high SNRs ($>$ 30 dB) $\sigma_{|\Delta A_{\text{HF}}|}$ is reduced by the inclusion of "a priori" information of respiratory frequency at the expense of an increasing $m_{|\Delta A_{\text{HF}}|}$. For lower SNRs ($<$ 30 dB), not only $\sigma_{|\Delta A_{\text{HF}}|}$ is reduced but also $m_{|\Delta A_{\text{HF}}|}$ decreases, being this improvement more notorious for lower SNRs. For a SNR of 5 dB an estimation error of 14%±11.5% against over 22%±17% is achieved when "a priori" information of respiratory frequency is included. For SNRs lower than 5 dB estimation errors are unacceptable in HRV analysis. The SNR in real HRV signals may

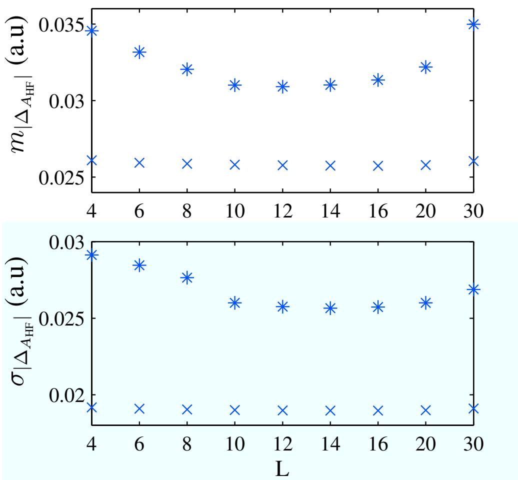

Figure 3. The mean $m_{|\Delta_{A_{HF}}|}$ (top) and SD $\sigma_{|\Delta_{A_{HF}}|}$ (bottom) as a function of L for a SNR of 20 dB with ('$\times$') and without ('*') inclusion of "a priori" information.

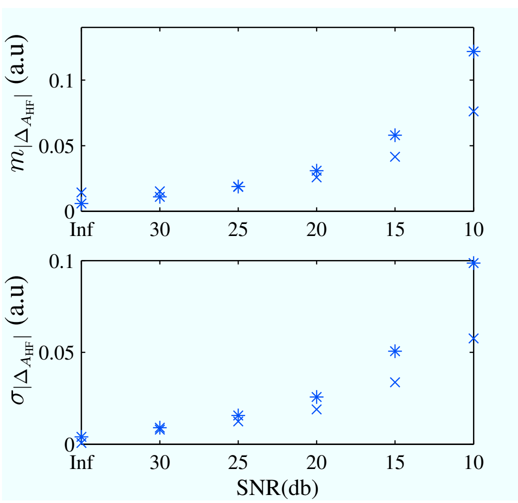

Figure 4. The mean $m_{|\Delta_{A_{HF}}|}$ (top) and SD $\sigma_{|\Delta_{A_{HF}}|}$ (bottom) as a function of SNR for L=12 with ('$\times$') and without ('*') inclusion of "a priori" information.

be below 25 dB, assuming typical values of HRV and misalignment errors over 1 ms, which makes the proposed technique work in the range of improved bias and variance. It should be pointed out that estimation errors using the method in 2.2 are dependent on the order of the backward linear prediction problem L, being usually lower in mean and higher in SD as L increases. However, estimation errors when "a priori" information of respiratory frequency is included are similar independently of the value of L, which may be an advantage when the order L is unknown and must be guessed.

4. Discussion and conclusions

In this work, the method proposed in [1], based on the automatic decomposition of the SWD using a LS approach, has been successfully applied to estimate the instantaneous frequency and amplitude of the LF and HF components of simulated stress testing HRV signals for SNRs over 10 dB. The inclusion of "a priori" information of respiratory frequency as the frequency of the HF component has been addressed. For high SNRs the inclusion of "a priori" information of respiratory frequency reduce the SD of the HF amplitude estimation errors at the expense of introducing a bias. However, for lower SNRs both the mean and the SD of the estimation errors are reduced.

It must be pointed out that even if the method has been designed and optimized for analysis of HRV during stress testing, its general features make it applicable in any HRV studies in which "a priori" information of respiratory frequency is available (i.e. patient monitoring, sleep studies, neurovegetative tests).

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