

# Design of New Reliable CFD-Based Estimation of Flow Rate: Early in-Vivo Results

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## Abstract

*In the present work a new approach for the blood flow rate estimate is discussed together with the performances results on both Computational Fluid Dynamics and in vivo datasets. In the mean time the comparison of this new method against the standard one used in clinics during catheterization in Doppler based flow rate estimate is provided showing important limitation of the latter and possible concrete enhancements using the new one proposed herein.*

## 1. Introduction

The correct knowledge of the blood flow rate ( $Q$ ) is a major issue in clinical practice. The elective approach for blood flow analysis is based on the Doppler technique which cannot measure directly the flow rate. The latter has to be indirectly estimated starting from the value of the maximum velocity ( $V_m$ ) and the value of the area of the vessel ( $A$ ). In particular from [1] an a-priori profile is usually assumed:

$$Q_D = A \times V_m / 2 \quad (1)$$

This equation involves a parabolic spatial profile for the velocity; in this sense this method has been also associated to a quasi-static hypothesis which can hardly hold true for blood. By means of about 200 computational fluid dynamics (CFD) simulations we designed a new formulation for the flow-rate estimate based on the Womersley number ( $W$ ): a dimensionless index used for quantifying the pulsatility of the flow (see [2]). In what follows we present how the new formula has been obtained and how it has been validated in a wide range of CFD models and under a wide range of fluid-dynamics physiological conditions (see [2] and [3]). Finally, the first possible approach to in-vivo validations of the new formula, as suggested and fully described in [4], is presented together with the very early results.

## 2. Methods

The general form of the new formulation is:

$$Q = g(V_m, W) \quad (2)$$

establishing a link between the mean velocity and  $V_m$  as a function of the Womersley number  $W$ . Parameters calibration have been performed using estimates of data in the form  $(V_m, W)$  obtained solving the Navier-Stokes equations, prescribing the flow rate boundary conditions without choosing a priori the velocity profile and thus avoiding any bias on the computed flow fields (see [5]), under a wide range of flow conditions in the range of  $W$ :  $2.7 < W < 15$ . The resulting formula, that will be named Womersley-based approach, can be written in the form:

$$Q_W = \begin{cases} g_1 & W \leq 2.7 \\ wg_1 + (1-w)g_2 & 2.7 < W \leq 3.1 \\ g_2 & 3.1 < W \leq 15. \end{cases} \quad (3)$$

Where:

$$g_1(V_m, W) = 0.5 \times A \times V_m \times (1 + a_1 W^{b_1}); \quad g_2(V_m, W) = 0.5 \times A \times V_m \times b_2 \arctan(a_2 W);$$

and  $a_1 = 0.00417$ ,  $b_1 = 2.95272$ ,  $a_2 = 1.00241$ ,  $b_2 = 0.94973$ ,  $w = (W - 2.7) / (W - 2.7) - (3.1 - 2.7)$ .

The performances of the new Womersley-based approach for blood volume estimate was tested and fully validated using again CFD in realistic geometries and under a wide range of physiological conditions (see [2] and [3]). In each case, we compared the percentage error on the flow-rate estimate using these two approaches (eqn. 1 and eqn. 3 respectively) given by  $Q_D$  and  $Q_W$ . Recently in vivo acquisitions of 2D phase contrast MRI of the descending abdominal aorta in an healthy young volunteer have been performed. As fully discussed in [4] the richness of the PCMRI data can be used twofold: in one hand to compute the effective value of the flow rate as integral of the velocity field across the vessel and on the other hand to evaluate the two estimate formula (eqn 1 and 3) by mimicking the Doppler acquisition and dataset. In figure

1 the overall scheme of the in vivo setup is shown.

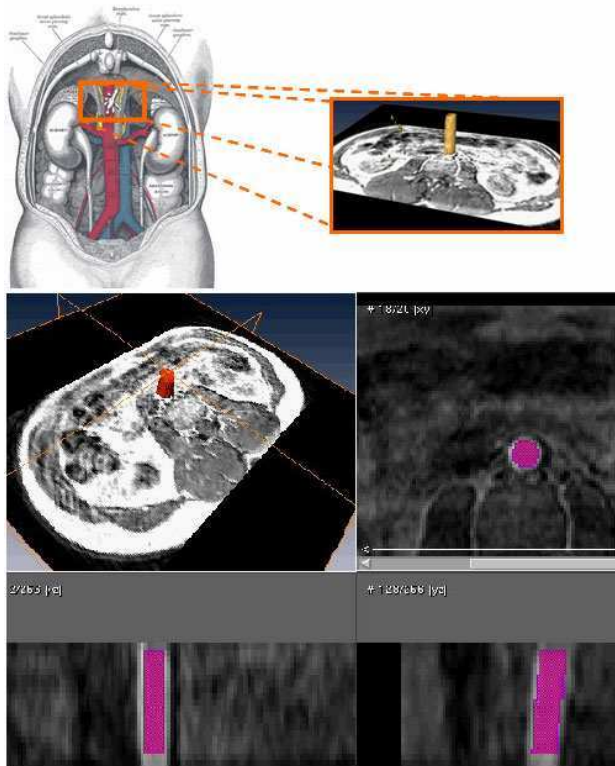


Figure 1. In-vivo experimental setup: the Phase Contrast MRI dataset.

### 3. Results

All the CFD results shows the inaccuracy (from 5% to more than 34%) of the parabolic approach using either systolic either diastolic flow wave forms and considering vessels diameters ranging from 2 mm to more than 20 mm. In figure 2 the overview of the percent of errors obtained using the parabolic formula (eqn. 1) on cylindrical domains at the peak velocity instant under different fluid dynamics conditions is plotted as a function of the Womersley number.

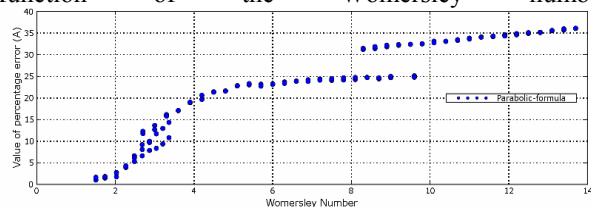


Figure 2. Percent of errors using equation 1 as a function of the Womersley number.

The error is increasing with the value of the Womersley number and is sensible to the shape of the flow wave form. Other CFD simulations on a carotid

bifurcation model, a total cavo-pulmonary connection (TCPC) and on three coronary Y-shaped bypasses shows the same behaviour at the peak instant (see [2] and [3]). In these CFD models using the Womersley-based estimate approach (eqn. 3) the error is considerably reduced.

Using the same indicators as in the results obtained using CFD modelling techniques, the validity of the two estimates methods (eqn 1 and eqn 3) have been compared in vivo using PCMRI technique (see [4]). Notably in all the 2D slices studied herein considering the peak velocity instant the order of magnitude of the error involved by the parabolic approach is of about 30%, instead the Womersley-based approach present a value of the error always below 5%. This is accordance not only with theory but also with all the CFD founding discussed herein. In fact in the in vivo dataset the mean Womersley number for the 2D slices is about 10 and the magnitude of the associated error applying the parabolic formula (eqn 1) is similar to the one predicted in the CFD models (about 30%). In figure 3 the plot of the 3D velocity profiles are shown in four selected slices and in three velocity instant across the peak one confirm these findings: the shape is considerably more flattened than a parabola sharing the same maximum velocity.

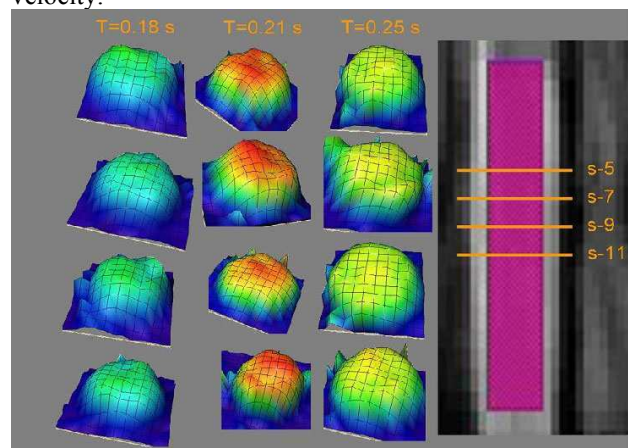


Figure3. Three-dimensional spatial velocity profiles obtained using PCMRI data in vivo.

### 4. Discussion and conclusions

The results obtained in this work show clearly the systematic incorrectness of the parabolic hypothesis used in daily clinical velocimetry analysis and suggest that a more accurate substitute accounting for the pulsatility nature of blood is needed. By applying the Womersley-based on both in-silico and in-vivo datasets in fact relevant improvements have been observed. This early results suggest that a more wide and complete in-vivo/in-vitro companion, based on the main idea undergoing the

validation described herein, has to be done in order to establish definitely the goodness of this approach for blood flow rate estimates.

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